On The Subject of Metaphor

Lyndon H. LaRouche, Jr.

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During the twenty-five-odd year reign of today's "New Age" cult, an ominous crippling of the U.S. individual's cognitive functions has been abuilding. This loss of mental capacity is presently affecting a growing majority among the under-fifty generations. Much of this damage is attributable directly to the multi-faceted influence of a modernist dogma which usually parades under such various names as "systems analysis," "linguistics," and "information theory."

Today, for example, rarely are pupils guided to reproduce, within their own minds, the Socratic experience of re-living the original discovery of crucial principles of scientific knowledge. Lacking the benefits of such once-traditional forms of secondary school learning in the subject matter of rigorous formal and synthetic geometries, for example, today's student would virtually never be able to attain an intelligible comprehension of even the bare fundamentals of physical science. Thus, today's modernist classrooms have been turned away from what is too often reviled as "authoritarian" teaching of conceptions; more and more, the modernist's "democratic" classroom and sterile textbook merely "provide information."

Similarly, a generation has passed since the time it was still fashionable to assess a pupil's progress in terms of that student's ability to apply prior learning to the effect of discovering, promptly, appropriate constructions of relevant solutions to unfamiliar problems. More and more, schools employ the "more efficient" practice, of degrading education to the rehearsing of pupils for passing computer-scorable forms of multiple-choice questionnaires.

These, and other enumerable applications of the pathological information theory doctrine, have brought upon us much of that widespread collapse of the individual victim's attention span which has occurred lately, accompanied by a correlated loss of the potential for those qualities of rationality which are associated with achievement in science and technology. That loss of scientific rationality is linked functionally to a parallel loss of personal capacity for comprehension and enjoyment of such once-respected fine arts as great music or the classical tragedies of Aeschylos, Cervantes,[1] Shakespeare, and Schiller.

Such observations pose the question: What makes an ostensibly innocent technical doctrine, such as information theory, so wickedly pathological in its social effects? The most efficient tack for exposing the answer to that question, is a more rigorous, Socratic definition of the fine arts term, metaphor. We signify "metaphor" as William Empson's *Seven Types of Ambiguity*[2]—for one—has identified it, as a phenomenon customarily associated with classical forms of poetry and drama. However, by "more rigorous," we should also show metaphor as the crucial feature of those thought-processes bearing upon the geometrical fundamentals of physical science.

That sets the task before us. So, without more foreword, to work.

I. What Is Metaphor?

In the case a literary construction points directly toward one object of attention, the ostensible subject, while uttering direct or implied reference to a different object, we have literary *irony*. Usually, to put the matter in its simplest terms, such irony is expressed usually in one of three forms: *comparison, hyperbole,* or *metaphor*. We may summarize fairly the most widely accepted academic view of this kind of ambiguity in the following terms.

The substitution of the name of another object for the customary name of the object in view, is traditionally considered in such academic climes as a matter of *symbolism;* that interpretation of these devices of irony is mistaken. Exposing this mistake of the academics leads

Rembrandt van Rijn, "Self-Portrait as the Apostle Paul," 1661 (detail) (SEE *inside back cover*).

us, in the relatively most direct way, to recognize that pathological fallacy of composition upon which Professor Norbert Wiener's information theory dogma is premised.

For this purpose, reference the domain of elementary geometry.

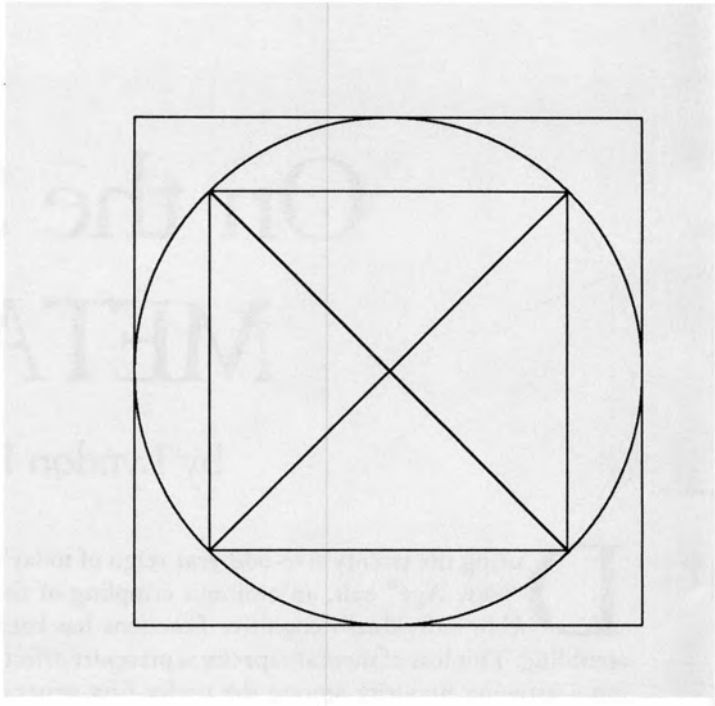
At an appropriate place in the secondary curriculum, the traditionalist teacher of secondary school geometry introduced the Pythagorean Theorem. The pupils of that class were guided to re-experience the mental act of original discovery by Pythagoras himself, thus to reconstruct a copy of that aspect of Pythagoras' creative mental processes within the mind of each of the pupils. This new existence within the pupil's own mind is itself an object of a special kind, a thought-object identified by the *metaphorical* name "Pythagorean Theorem."

The crux of this example is the fact, that the thought-object associated with the metaphorical name "Pythagorean Theorem," is neither an object of the outward senses, nor an object which can exist explicitly within any medium of communication.[3]

In this location, our primary argument is focused upon another example from the realm of synthetic ("constructive") geometry, Nicolaus of Cusa's revolutionary insight into the paradoxical Archimedes' Theorems which treat the subject of squaring the circle.[4] This example shall serve us here, henceforth, as the model reference for an initial, more rigorous definition of classical metaphor. It is also a point of reference, therefore, for treating Wiener's fundamental fallacy.

Cusa proved, early in his adult life, that no curved line can be generated by means of joining together many very small straight lines. This proof led directly to the seventeenth-century discovery of the principle of physical least action, that all physical functions are of a species termed "non-algebraic" (or, "transcendental"), rather than arithmetic or algebraic. This, Cusa's referenced discovery, has the positive relevance, of being the continuing point of origin, and the mathematical cornerstone of the past five hundred years' birth and development of modern physical science.

The additional consideration to be stressed, is that this particular discovery by Cusa typifies all cases of creative forms of fundamental discovery in both science and the fine arts. That is to emphasize: solutions to real problems for the case that there exists no solution solely by means of deductive methods of argument. Those non-deductive solutions, solutions by methods which cannot be represented explicitly by any linear medium, such as communications media, typify the class of thought-objects to which belong the pupil's reliving of Pythagoras' discovery and of Cusa's discovery of an isoperimetric *species* of circular action absolutely distinct from the



species of all possible linear functions.

It is thought-objects of that class which are the center of our attention here. It is the use of *communicable* arrays of *names* to identify members of that special class (species) of thought-objects, which we hold forth here as the proper form of illustration of the principle of metaphor.

We shall return to the case of metaphor in fine arts practice, after we have explored the definition of metaphor in the practice of physical science.

Squaring the Circle

Cusa reworked the four theorems of Archimedes on the subject of squaring the circle, by constructing a square whose area is equal to, and derived by construction from the preceding construction of a given circle. This assignment might be interpreted in two alternative ways. The student of algebra would wish to construct a square whose area (a^2), differs by no more than a negligible amount from that of a given circle, πr^2. The student of constructive geometry would demand that we accomplish this algebraic result by no means other than a strict, explicitly, exclusively geometric argument. Cusa focused upon the latter, geometric requirement.

From the secondary geometry classroom: the method for estimating the area of a square approximately equal to that of a given circle, is this. Simultaneously inscribe and circumscribe a pair of regular triangles, or squares (SEE Figure 1). Next, by halving angles, by construction, repeatedly double the number of sides to, for the squares, some number equal to 2^n. Take the average of the areas of the two polygons; estimate the value of π, the ratio of the circle's perimeter to its diameter, by dividing the average area of the two polygons by the factor of r^2 (the

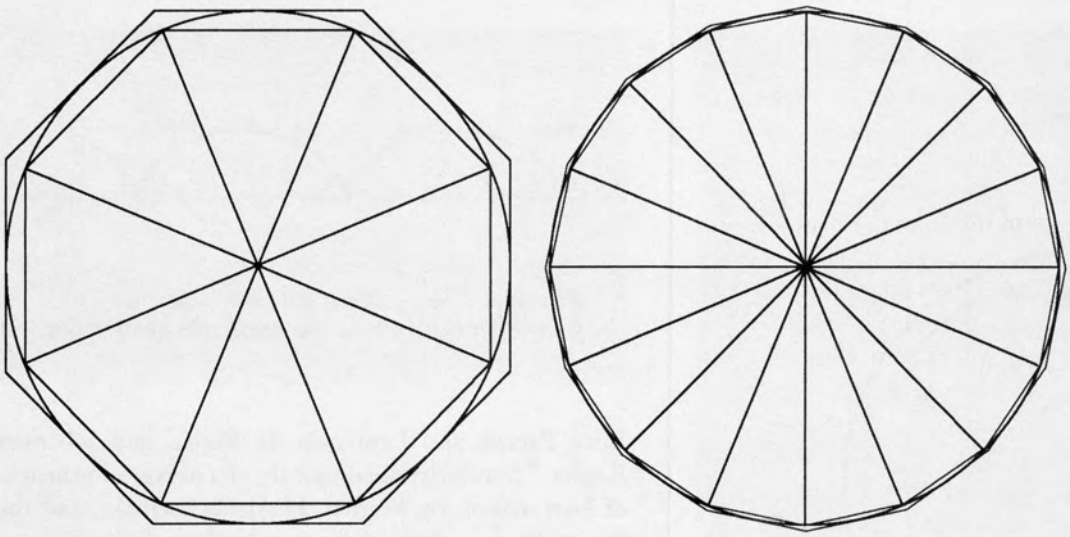


FIGURE 1. *Estimating the area of a square approximately equal to that of a given circle, as the average area of two regular polygons.*

square of the radius). Thus, for $n = 8$, π is estimated at approximately 3.1416321; for $n = 16$, the estimate for π is a much better approximation, 3.1415927.

Nonetheless, there is a very stubborn and profound paradox in this apparent algebraic success. This leads us to Cusa's discovery, and, from that point of origin, to the seventeenth-century discovery of a differential calculus of non-algebraic, least-action functions, by Leibniz and the Bernoullis.[5]

Admittedly, by the indicated method of estimated averages of the two regular polygons, we could estimate the square area of the given circle to any decimal position, according to the given algorithm. Ask the question: Does the perimeter of the inscribed polygon become ultimately congruent with the bounding circumference of the circle? With that question, a devastating paradox confronts us. Take the illustrative case, that $n = 16$; look at a region of the circular circumference of one minute—one-sixtieth (1/60) of a degree. There are slightly more than 182 angles of the inscribed polygon within each degree of measurement of the circle's perimeter—slightly more than three per minute (SEE Figure 2). At the far extreme of $n = 256$, there would be approximately 3.216×10^{74} angles of the polygon for each degree of circumference. At a mere $n = 112$, for a circle of 1-centimeter radius, the distance along the circle's circumference between angles would be approximately 1.21009×10^{-33} centimeters, approximately the limit of a Planck distance in quantum microphysics.

Thus, the more nearly perfect our estimate of the circle's square area, the greater the degree of *ontological* difference between the circumference of the circle, as a geometric species of action, and the perimeter of our developing 2^n polygon, as a second species. The more

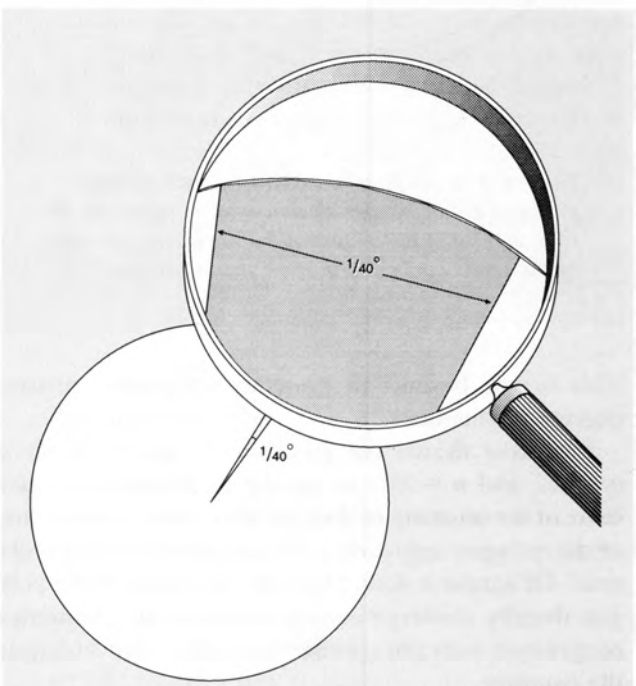


FIGURE 2. *An inscribed polygon of 2^{16} (65,536) sides may* seem *to closely approach a circle in area. But there is a devastating paradox in the seeming equivalence. There are slightly more than 182 angles of the inscribed polygon within each degree of circular arc.*

nearly the polygon's perimeter approaches the trajectory of the circle's circumference, the greater the frequency of discontinuities in the polygonal perimeter, and, therefore, the greater the difference in species of geometric form between the circular and polygonal perimeters.

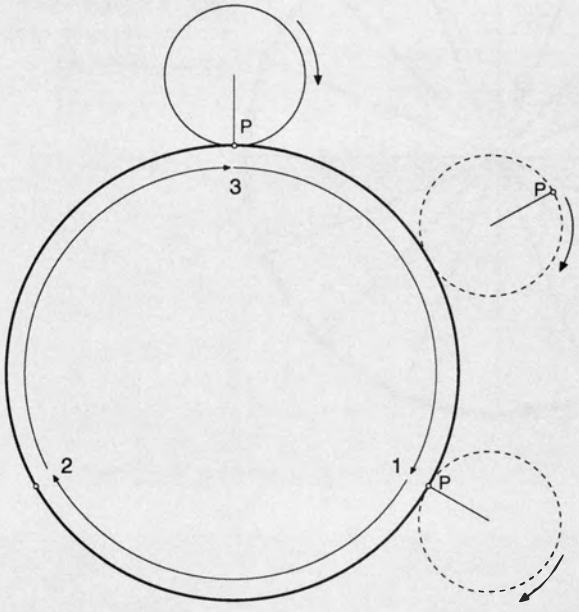


FIGURE 3. *A circle rolled against another circle produces either integer or non-integer values by the time it returns to its starting point. Here, the radius of the small circle is one-third the larger one.*

This is true beyond all presently imaginable physical degrees of smallness.[6]

We have drawn the paradox out to the limits of $n = 112$ and $n = 256$, to impart a relevant emotional sense of the intensity of that paradox. Does a square area of the polygon approach approximation of the circular area? Of course it does. Does the perimeter of the polygon thereby converge asymptotically upon geometrical congruence with the circular circumference? No, quite the contrary.

The paradox so adduced from Archimedes' theorem, is also exemplary of the proper posing of the problem underlying all among those scientific discoveries which have more than a merely crucial significance for existing scientific knowledge. The solution to this paradox has what is best termed a *unique* quality of fundamental importance for all facets of scientific knowledge in general.

These paradoxes are all of the type exemplified by Plato's *Parmenides* dialogue, on the all-encompassing topic of "the One and the Many."[7] Leading subsumed cases of unique discovery include each and all of the successive treatments of the "Platonic solids" by Plato,[8]

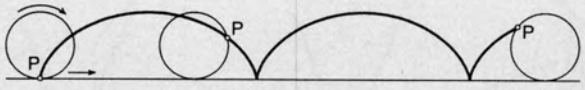


FIGURE 4. *The cycloid is the curve traced out by point P on the circle, as the circle rolls along a line.*

Luca Pacioli and Leonardo da Vinci,[9] and Johannes Kepler.[10] Similarly, the discovery of a universal principle of least action, by Fermat, Huygens, Leibniz, and the Bernoullis,[11] is derived from preceding discoveries including both the isoperimetric principle and the implications of the Platonic solids. Examine the following crucial features of those interconnections.

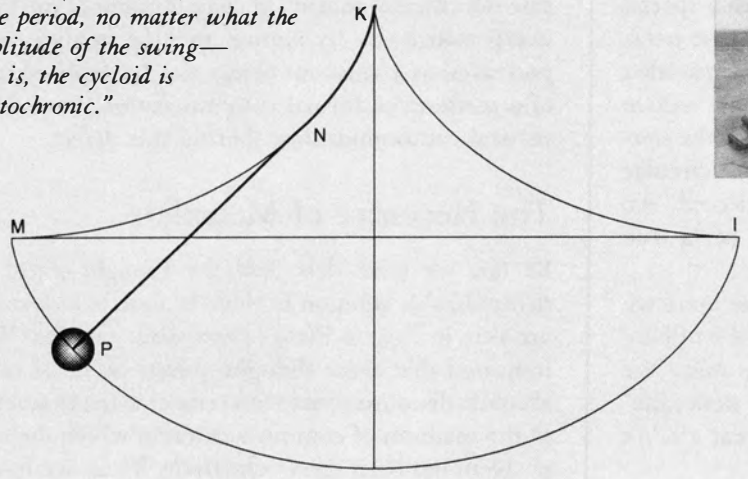
The Isoperimetric Principle

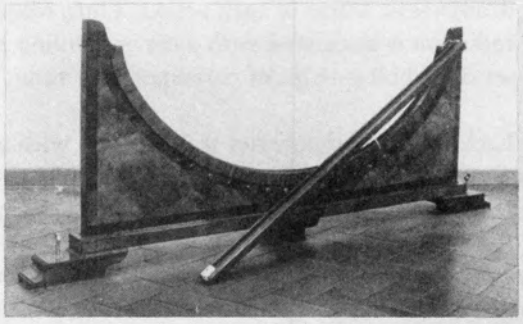
The application to the squaring of the circle of that method of addressing such a paradox which is exemplified by Plato's *Parmenides* dialogue, yields essential results which are the common feature of each and all of the solutions for a series of the most fundamental scientific discoveries of the period from c.1440 A.D. through c.1700 A.D.. For reasons to be considered, these features are all presented from a negative standpoint:

1. Circular action is a distinct geometrical *species* of action in space-time, the which cannot be derived from any *species* of linear construction. No *positive* definition of circular action may be employed, if that definition specifies in any part a required *point* or piece of *straight line* (such as a radius).
2. Circular action is defined simply (negatively) as the *least action* of closed perimetric displacement required to subtend the relatively largest area. (Thus, the Fermat-Huygens-Leibniz-Bernoulli principle of least action is already implicit, "hereditarily," in Cusa's discovery.)
3. Circular action, because *closed* (SEE Figure 3), is a form of continuous extension (continuous manifold) which contains its own metrical characteristic: counting in cycles and parts of cycles. A linear continuous manifold contains no inherent metrical quality which is not supplied to it by the external bounding imposed by a higher geometrical species of continuum.
4. Circular action bounds externally, and thus deter-

FIGURE 5. *The least-action principle embedded in cycloid functions.*

In his 1673 On the Pendulum Clock, *Huygens demonstrated that a pendulum made to follow the path of a cycloid (curve MPI) will have the same period, no matter what the amplitude of the swing—that is, the cycloid is "tautochronic."*





A ball rolling down a cycloidal track will reach the bottom in the same time, no matter where on the track it is released. Later, Johann Bernoulli demonstrated that the cycloid also has the property of a "brachistochrone"—it is the least-time pathway. (Model in the Museum of the History of Science, Florence, Italy.)

mines all linear species of constructions.

This is underlined by the paradoxical features of the stated case for the relative uniqueness of the five Platonic solids as stated by Plato, Pacioli, Leonardo, and Kepler. That is made general by the development of treatments of the cycloids (SEE Figure 4), from the work of Christiaan Huygens onward: all physical and arithmetic functions are properly stated in nothing less than terms of those non-algebraic functions which are derived "hereditarily" from the germ of the cycloid, and from the least-action principle embedded in the cycloid functions (SEE Figure 5). This is first demonstrated in physics, from Leonardo da Vinci through the work of the Bernoullis, for light (propagation of electromagnetic radiation) and hydrodynamics.

5. The additional crucial feature of circular action, is that it defines our universe in terms of both negative and positive curvatures, with the demonstration that negative curvature predominates. This point is summed up rather neatly in Johannes Kepler's 1611 booklet, *On the Six-Cornered Snowflake*.[12] The snowflake is a non-living process determined by the function of positive curvature in determining the close packing of spherical bubbles.[13] The negative curvature of the interior of each and all bubbles determines structures "hereditarily" cohering with the five Platonic solids, and, thus with the harmonic orderings cohering with the Golden Section of the circumscribing sphere's great circle.[14]

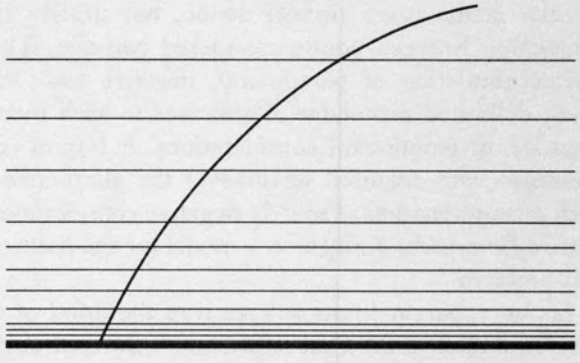


FIGURE 6. *The least-action principle. Johann Bernoulli proved (1697) that the path of a ray of light passing through a medium of constantly increasing density, will be a cycloid.*

The universe can be considered as everywhere superdensely packed with spherical bubbles of all imaginable radii, as the unique, bounding characteristic of generalized "non-algebraic" function shows this to be necessarily the case. By the close of the seventeenth century, it was implicitly demonstrated (SEE Figure 6), that this bubbly universality of the least-

action principle is otherwise characterized by the combined notions of electromagnetic least action and hydrodynamic forms of such action. Thus, frequency of radiation is associated with a corresponding resonant set of bubbles—e.g., of corresponding radii.[15]

Each of these discoveries is associated with a special kind of paradox, which might be termed "a true paradox." In the instance of squaring the circle, the paradox is, that the more successfully we estimated the square area of the circle, the more extremely we proved the *non-congruence* of the polygonal perimeter with the circular circumference. "The more we appear to succeed, the more we truly fail," might be a fair image of "a true paradox."

So, in the case of the five Platonic solids, the more we attempt to circumvent the limitation identified by Plato, as did Archimedes, Pacioli, and so on, the more we understand the germinal uniqueness of the dodecahedron, and of the Golden Section of that great circle's negative curvature.

By the close of the seventeenth century, the successive work of Huygens, Leibniz, and the Bernoullis on the tautochrone/brachistochrone problem for isochronism and for light, had shown implicitly that all possible action in our universe must conform to multiply interacting circular action upon circular action, not straight line interaction between points considered pairwise. Thus, the accumulation of paradoxical, negative considerations, delimited acceptable alternatives to such merely negative, or paradoxical considerations. A leap of consciousness was required to discover the alternative to such a concatenation of merely negative considerations. Plato's *Parmenides* dialogue is a model for the nature of this problem.

So, an apparent solution leaps into the mind of the successful discoverer. That solution, as a thought-object, cannot be directly depicted in terms of communications media available. Thus, if it cannot be communicated explicitly, how might we know whether the newborn thought-object were valid, or not? There are two conditions which prompt us to recognize such a thought-object as valid. First, it satisfies all of the negative conditions associated with the relevant paradox. Second, it goes beyond those negative requirements, to enable us to generate efficient hypotheses, reaching by these means into realms which were unattainable for us without the aid of these new thought-objects.

By signaling both the negative preconditions of a hypothesis, and also, similarly, describing efficient new constructions derived from the new thought-objects, we communicate to our own and other consciousness the formal proofs of the thought-object's validity. Thus, we may be relatively certain, that the thought-objects so generated by different, communicating intellects are congruent thought-objects.

Therefore, by citing the name of the thought-object among those who share its possession, we may communicate the efficient sharing of consciousness of the thought-object which, by its nature, may be neither explicitly portrayed as a sensuous object, nor be depicted in terms of a medium of formal communication, formal mathematical communications sharing this defect.

The Necessity of Metaphor

So far, we have described the thought-object as the demonstrable solution to those unique paradoxes which are akin in *Type* to Plato's *Parmenides* paradox. We have indicated that these thought-objects occur as relatively absolute discontinuities with respect to the characteristics of the medium of communication in which the relevant problem has been stated *negatively.* Thus, we have indicated, the thought-object itself cannot be depicted explicitly within the domain of the communication medium. However, the reference to such a thought-object can be recognized by a hearer whose mind contains the sibling of that same thought-object.

In a classical humanist form of secondary school education, most emphatically, the emphasis is on presenting the pupils with the most important among the unique and other relatively elementary discoveries in the entire historical sweep of the advancement of civilized knowledge. It is desirable that original sources be used whenever they are both available, and in a form suited to that stage in maturation of the pupil's powers of comprehension. Otherwise, only if such suitable primary sources are not available, we should rely upon paraphrases which effectively and fairly state the true paradox associated with that original discovery.

This form of classroom introduction to such original sources has a required order, as the ordering of Euclid's *Elements* of geometry illustrates, from a formalist standpoint, the notion of a choice of such an order. Secondly, the ordering is determined by the consideration, that mastery of one discovery is virtually prerequisite for the comprehension of a successor in that series. The sound secondary curriculum teaches geometry and the plastic arts, as *the domain of visual experience,* as, in parallel, the student concurrently learns language, literature, and music—*the domain of hearing.* The historical order internal to the sciences of vision and hearing provides a virtually indispensable concomitant to the study of the rise of the European (Christian Humanist) Renaissance

of the fifteenth century, out of ancient and medieval history, and upon that foundation, the study of global post-Renaissance history.

Several most important effects are fostered by such a classical humanist form of secondary education.

In each case, first of all, the pupil replicates an original discovery. Within the student's own intellect, there is approximately a replication of the mental processes of that creative discovery which was experienced earlier by the original discoverer. Later, the pupil experiences another such crucial discovery, by an original source who depended, in turn, as the student does, upon the prior of these two original sources considered. So, it continues. So, in respect to mathematics and physical science, for example, the pupil's mind is populated, in effect, by a growing number of such past historical personalities of science, to the effect that the pupil not merely imagines these persons, as if they were merely characters in some story, but knows each as a living, thinking person, through the replication of some of the creative processes of each within the pupil's own mental processes.

Functions of 'Discontinuity'

In that illustrative case from geometry which we have treated thus far, the Platonic form of paradox embedded within an Archimedean estimate for the squaring of the circle, it is shown, that even far, far beyond the already logically meaningless case of an hypothetical regular polygon of 2^{256} sides, there remains a distinct, intelligibly measurable gap between the relatively lesser area of each and all inscribed regular polygons and the marginally greater area of the relevant circle. The persistence of the discreteness of that gap, persisting beyond all limit of such extension, is a model for a simple type of mathematical discontinuity. It is not the magnitude of the gap, which is this discontinuity; the discontinuity is the fact of the persisting, *transfinite* discreteness of this gap, however tiny that persisting gap were to become.[16]

Examine that class of simple types of discontinuity from a subsuming vantage-point. Explore, in this way, the nature of those mental existences which we have identified as thought-objects. Consider formal theorem-lattices.[17]

The short definition of a deductive theorem-lattice is provided in the following three, complementary statements. Given, any constant, integral set of deductive axioms and postulates:

1. No consistent theorem derived from that set of axioms and postulates states anything which was not already implicit in that set of fixed underlying assumptions.
2. Any theorem of this lattice, which is constructed to represent an experience, will project upon such representation nothing but the ideas of the *ontological qualities* and *behavioral potentialities* already implicit in the latter's underlying, integral set of axiomatic and postulational assumptions.
3. Any demonstration which refutes a single deductively consistent theorem of such a theorem-lattice, refutes axiomatically the choice of underlying integral set of axioms and postulates upon which each and all possible hypotheses or theorems of that theorem-lattice depend.

Thus, for example, to the degree which the intended development of the mathematical physics of a Descartes and Newton is intended implicitly to perfect itself as a deductive theorem-lattice, the development of that physics has the combined form of expanding the number of theorems, while perfecting the deductive consistency of the expanding theorem-lattice as a whole. When nature itself manifestly denies, even in a single instance, what is shown to be a consistent theorem of that lattice, the entire lattice's underlying *an* integral set of axioms and postulates must be altered. The alteration must remedy the disagreement with nature in that one crucial instance, but without producing experimentally invalid forms of other theorem types.

Let us recognize that principle of axiomatic consistency of a deductive theorem-lattice by the sometimes employed term, "hereditary principle." Let us represent a successful, generalized, successive, step-wise improvement over deductive theorem-lattice *A,* by the series *A, B, C, D, E,* . . . .[18] The difference between any two adjacent terms of this series, is some change in the underlying, integral set of axioms and postulates of the predecessor term. Thus, for reason of that change, no deductive consistency exists between any one term of that series, and each and all other terms of that same series. This gap separating each term of that series from each among all the other terms, and doing this with deductive absoluteness, is a discrete discontinuity in the same broad sense as the gap separating the linear generation of the constructions of a regular polygonal perimeter from the different, isoperimetric quality of the relevant, inscribing circular action.

In the simple case of squaring the circle, we address a single object, that circle. *The germ of so-called "non-algebraic," or "transcendental" functions is already there, in Cusa's treatment of this paradox;* but, we must take additional steps to see it clearly. We must recognize a principle, integral to all competent mathematics, intrinsic to the notion of an isoperimetric circular action;

we must recognize the pervasiveness of a non-algebraic principle, which Gottfried Leibniz *et al.* named *analysis situs*. The cycloid is the best vantage-point for a secondary classroom treatment of these matters.

Roll a relatively very small circle (r) along the outside perimeter of a relatively extremely large circle (R). The result is that the perimeter of the very large circle, R, appears relatively almost a straight line.[19] At the start of this roll, the perimeter of circle r touches the perimeter of the larger circle at point P_0; to this corresponds point p on the smaller circle's circumference (SEE Figure 7). Roll circle r clockwise, making a series of points of tangency of p on circle R each time the rotating point p again touches the perimeter of R. Thus, between points P_0 and P_1, the trajectory of p forms a curved line, a cycloid, approximately that of Roberval[20] or Christiaan Huygens[21] (SEE Figure 8).

Extending the class' study of cycloids a bit, we meet the fact, that there is a different result if circle r is rolled upon the inside, as opposed to the outside of the perimeter of R. Take the cases that $R = 2r$, and $R = 3r$, and $R = 4r$, and $R = 5r$ (SEE Figure 9). These constructions draw our attention to the fact that there is an important functional difference between the *positive* curvature of the exterior of a circular perimeter, and the *negative* curvature of the interior of that perimeter.

Then, we follow Huygens through his treatments of the *tautochrone* and *involute-evolute* constructions (SEE Figure 10).[22] Together with Huygens, Leibniz, the Bernoullis, *et al.*, we discover several things which are crucial for all valid developments in mathematical physics, from approximately 1700 A.D. onward, to date, things bearing directly upon these principles of metaphor.

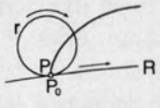


FIGURE 7. *The cycloid is a vantage point for studying non-algebraic functions. Small circle r is rolling along the perimeter of large circle R.*

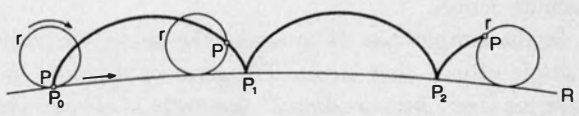


FIGURE 8. *A cycloid is traced out as point P on circle r rolls from P_0 to P_1, and from P_1 to P_2.*

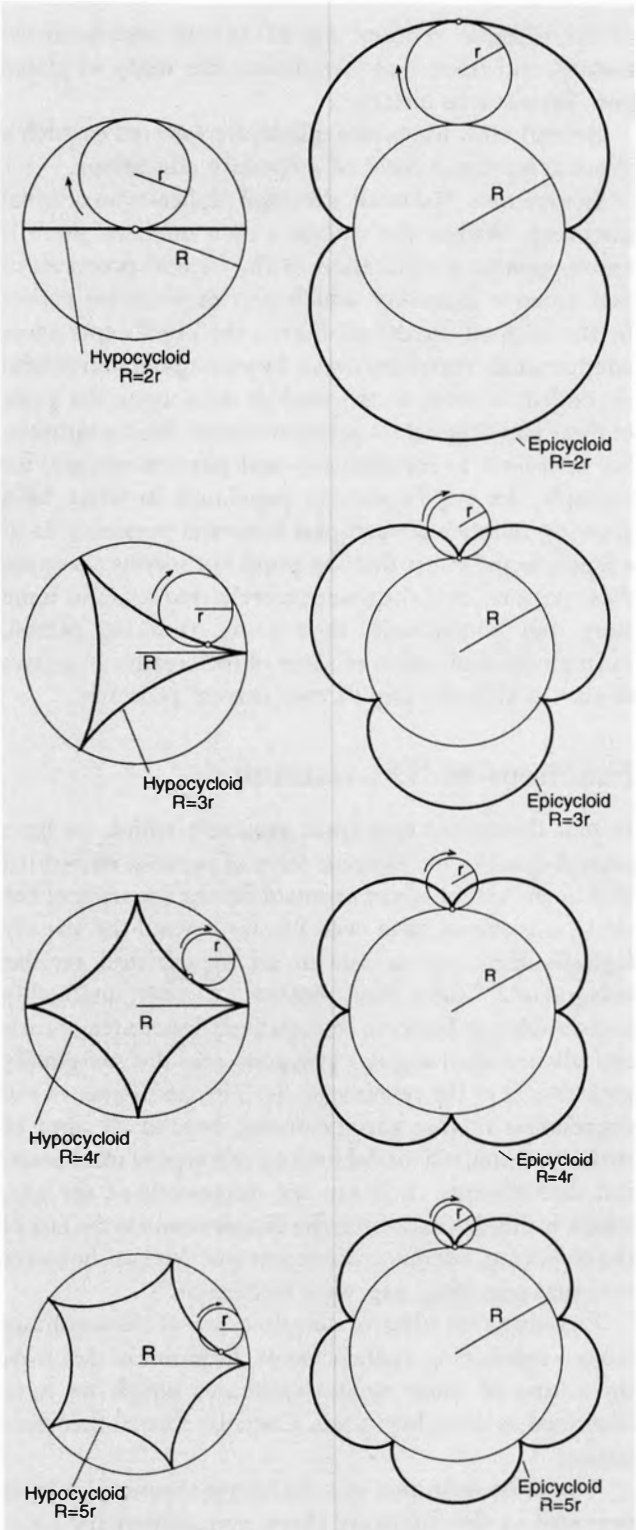


FIGURE 9. *Positive and negative curvature. The figures derived by rolling a circle on the* interior *of a larger circle (hypocycloids) are of a different species than those produced by rolling it on the* exterior *of the same circle (epicycloids).*

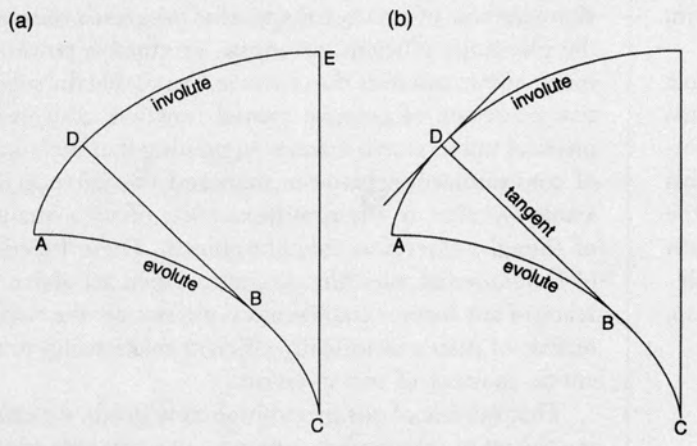
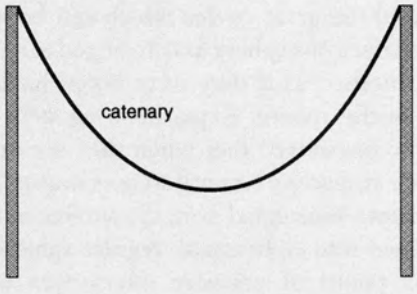


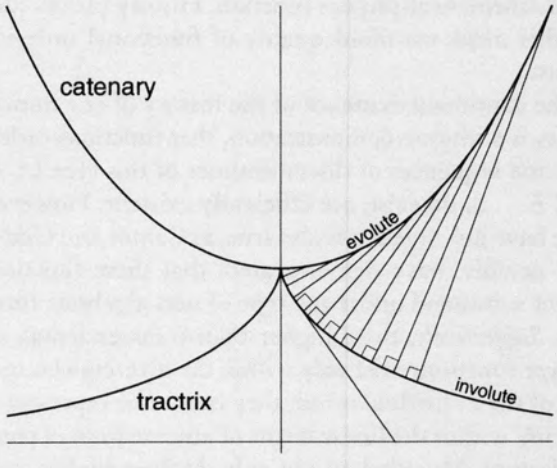
FIGURE 10. *In* On the Pendulum Clock, *Huygens presents involute-evolute constructions as follows. (a) "If a string or flexible line is understood to be stretched around a line curved in one direction, and if one end of the string remains fixed while the other end is pulled away, such that the freed part of the line always remains taut, then it is clear that some other curve is described by this end of the string. This latter curve is called the* involute. *The curve around which the string had been stretched is called the* evolute." *(b) "Every straight line tangent to an evolute meets the involute at right angles." In the case shown, the involute of a cycloid is itself another cycloid.*

No student should be graduated from any secondary school, unless he or she has assimilated the treatments of cycloid, tautochrone, and involute-evolute relationships as put forth in Huygens' work on these subjects. Without that, and without the mastery of the tautochronic principle of least action for refraction of light as Leibniz and the Bernoullis set this forth during the 1690's (SEE Figure 6), there could be no competent grounding of the student in the barest prerequisites of as much as uttering the term "modern physical science." (How many science and engineering professionals today have met that requirement?)

In the very simplest case, the simple cycloids, *non-algebraic functions,* represents that class of functions derived, "hereditarily," by performing circular action upon circular action. This is then extended, to indicate circular action upon that result, and so on more or less indefinitely. This is extended, in turn, by Huygens, Leibniz, the Bernoullis, and by Gaspard Monge later,[23] to include those derived constructions obtained in the classroom by winding and unwinding taut threads, the so-called *evolutes* and *involutes* (SEE Figure 11). This includes the class known as *envelopes.*[24] We must include, as a matter of strict principle, the notion of *analysis situs* introduced by Leibniz.[25] We must include, retrospectively, Kepler's work on applications of the distinction between *positive* and *negative* curvature,[26] doing this from the standpoints of both elementary *analysis situs* and the treatment of

FIGURE 11. *Illustrating an involute-evolute relationship: The endpoint of a taut thread wound and unwound on a catenary (the curve formed by suspending a rope between two points), traces out a curve called a "tractrix." The* normals *(perpendiculars) to the tractrix are tangent to and* envelope *the catenary.*





negative curvature in the Riemann-Beltrami counter-attack upon the intertwined hoaxes of Clausius-Kelvin, Helmholz, and Maxwell.[27]

Short of Cantor's *aleph* (ℵ) transfinites,[28] all possible functions in mathematical physics, including problems of number theory, are non-algebraic, essentially geometrical functions of this extended *transcendental* form. This specific point will bring us soon here to the case of the problems posed by the widespread influence of such related hoaxes as "information theory," "systems analysis," and the "linguistics" of Russell, Korsch, Carnap, Harris, and Chomsky.[29]

Metaphor and Function

With the consideration of the indicated series, *A, B, C, D, E, . . .*, we define an ordered sequence of these terms. The "variable" of this ordering is not those terms themselves, but, rather, the discontinuities separating each term from each of all the others. These discontinuities are the point of intelligible access to the relative ontological nature of the classes of thought-objects to which we have referred above.

Consider Cantor's *alephs*. We have $\aleph_0$, $\aleph_1$, $\aleph_2$, and so on. These *alephs,* so ordered as in any sequence, form a *manifold.* This manifold is of a Cantor *Type;* this *Type* is ontologically of the quality of discontinuity separating each of $\aleph_0$, $\aleph_1$, etc., from all others.

This manifold and its *Type* cannot be reduced to any notion of *function* which is consistent with our use of the term "function" to denote a class of geometrical, non-algebraic, or transcendental functions. Yet, the *aleph*-manifold, with its many alternative orderings, is defined by a *typical quality* of all such orderings. That implies a notion of "function," although in no conventional sense of a mathematical-physics function. History proves such a higher *aleph*-manifold quality of functional ordering to exist.

The continued existence of the history of our human species is a unique demonstration, that functions ordering some sequences of discontinuities of this type (*A, B, C, D, E, . . .*), do exist, are efficiently existent. However, as we have just noted, it is also true, as Cantor and Gödel, most notably, have demonstrated, that these functions are not subsumed under the type of non-algebraic functions. *Subjectively,* these higher-than-transcendental, *aleph-type* functions exist only within the sovereign boundaries of the individual mind; they cannot be represented *explicitly* within the linear terms of any medium of communication. Nonetheless, not only do these higher, *subjective* functions exist; they are demonstrably efficient causal agencies in our physical universe.

The historical fact, that these higher functions are characteristic of successful scientific progress's raising of the physically efficient, *per-capita,* productive powers of man's command over the universe, shows that the subjective processes, of creative-mental function, address the physical universe in a manner suggesting that such forms of communication between man and the universe *as a whole* are akin to the communication of such qualities of thought-objects as thought-objects. These functions of fundamental scientific progress, which act above the reach of any formal mathematical physics, are the characteristic of man's historically efficient relationship to scientific mastery of our universe.

That outline of our proposition now given, we examine the set of relationships among *names, thought-objects,* and *our universe.* Let us speak of three domains. *First,* the domain of thought-objects, within the sovereign bounds of the individual's mental-creative life. *Second,* the relevant plane of sense and communication media. *Third,* within the physical universe, behind the superficiality of sense-experience, an underlying governing agency of principle, which controls the lawful behavior of the universe, and which will "recognize" certain of our changes in forms of actions with a favorable response.

Reference Figure 12. We have person *A,* a secondary-school teacher, and also an experimenter. We have person *B,* a student, and an observer of the experiment being performed. There is the experimental subject, *X. A* acts upon *X.* Student *B* observes *X,* and also observes *A*'s actions upon *X* throughout the experiment. *A* communicates, reciprocally, with *B,* a communication which precedes and accompanies the experiment, and which continues after the experiment's completion.

A, beginning from a thought-object in his own mind, provokes the replication of that thought-object within the mind of student *B.* This occurs through the method of *Socratic negation,* as applicable to a case which meets the requirement to be *a true paradox.* Consider an example, related to the Cusa isoperimetric paradox, which illustrates this phase of the transactions among *A, B,* and *X,* in this illustration; consider the proof of *the uniqueness of the five Platonic solids.*[30]

Take three great circles which can be moved about on the surface of a sphere and arranged at any inclination one to another, as if they were hoops having the same radius as the sphere. Experimenting with such hoops, it will be discovered that when they are arranged such that their respective circumferences mutually divide one another into four equal arcs, the surface of the sphere is partitioned into eight equal, regular spherical triangles. The six points of pairwise intersection of the hoops will be found to form the vertices of an octahedron

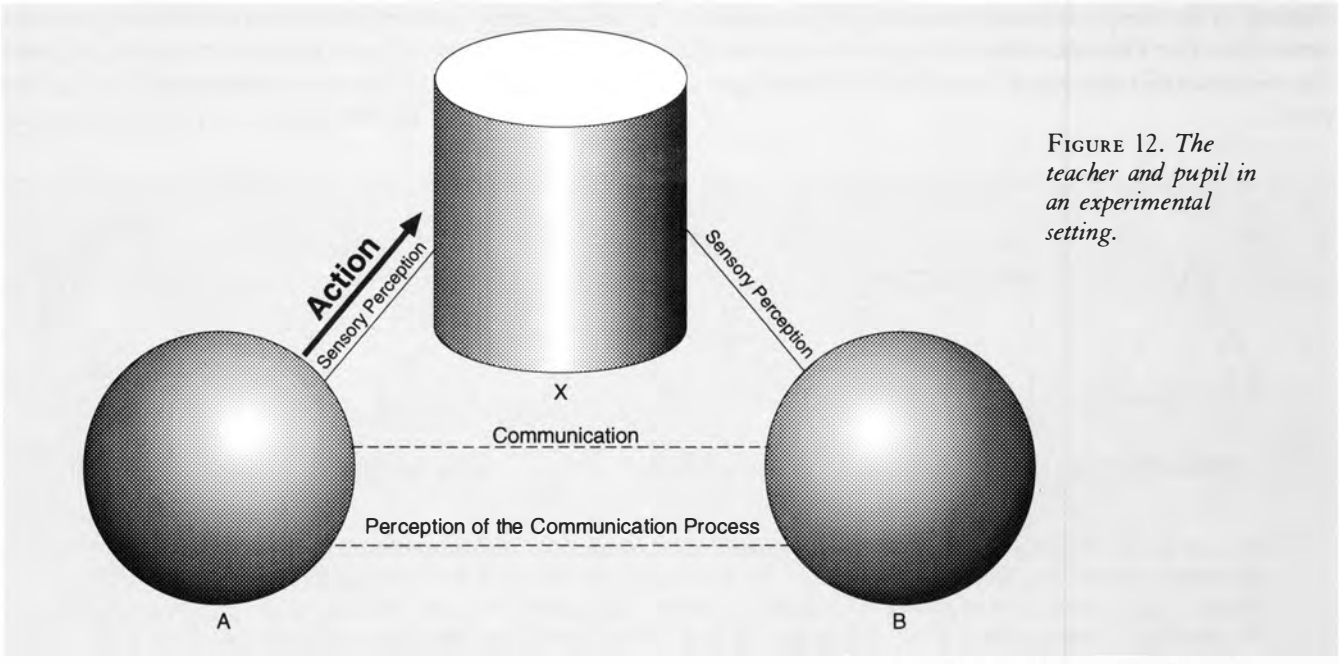


FIGURE 12. *The teacher and pupil in an experimental setting.*

(SEE Figure 13).

Do the same for four and six hoops. For four hoops, the pairwise intersection occurs at twelve points, coinciding with the twelve vertices of a cuboctahedron (the truncation through midpoints of edges of the cube or octahedron). The surface of the sphere is thus partitioned into eight equal and regular spherical triangles and six equal and regular spherical quadrilaterals. Each great circle is divided by the others into six equal arcs.

Using six hoops, thirty points of pairwise intersection result, forming the vertices of an icosidodecahedron (the truncation through midpoints of edges of the icosahedron or dodecahedron). The surface of the sphere is partitioned into twelve equal and regular spherical pentagons and twenty equal and regular spherical triangles. Each great circle is divided by the others into ten equal arcs.

It can then be proven that there are no other partitions of the sphere resulting in the division of the great circles into equal arcs. From the limiting case of six hoops, which permits the construction of twelve pentagonal faces, is demonstrated the primacy of the dodecahedron, and relative uniqueness of the five Platonic solids.[31] From the six-hooped figure containing dodecahedron and icosahedron, the cube, octahedron, and tetrahedron may be readily derived.

The Golden Section may then be conveniently demonstrated as the ratio of radius to chord on the dodecahedron formed by inscription in each of the six great circles, or, alternatively, as one of the many well-known internal relationships of the pentagon, formed by projection of the spherical pentagon onto a plane. In either case, the derivation of this ratio from the construction upon the sphere is to be stressed, rather than derivation from a pentagon or pentagonal division of the circle, presumed as given or constructed by algebraic artifice.

This approach has shown several points which are of crucial importance:

1. The necessity of deriving these regular polyhedra from regular spherical triangles, quadrilaterals, and pentagons is shown. This correlates with our earlier study of the paradoxical effort to square the circle. The construction of the polyhedra is bounded *externally* by *spherical action.*
2. That, only regular division of the sphere's surface by the factors three, four, and five succeeds. Thus, the dodecahedron corresponds to the *upper limit* of construction, since it is derived from fivefold division. No regular polyhedron of hexagonal sides, or larger, is constructible.
3. That all five regular solids are derived from the construction of the pentagonal-sided dodecahedron.

A strong indication of this is the following view of harmonic orderings cohering with the Golden Section.

The customary classroom and related practice, is to explain the construction of the Golden Section as necessary for the construction of the regular pentagon. This seemingly innocent practice has contributed to the circulation of much nonsense, nonsense which is avoided if

the Golden Section is situated directly within a proper reading of the simple construction-proof of the uniqueness of the five Platonic solids. Turn, for illustration of the point, to reference again Pacioli's *De Divina Proportione*.

Pacioli, Leonardo da Vinci, *et al.,* showed that, on the scale of direct sensory observation of ordinary processes, all living processes have an harmonic ordering of growth and morphology of function which coheres, as a *Type,* with the Golden Section; whereas, all non-living pro-

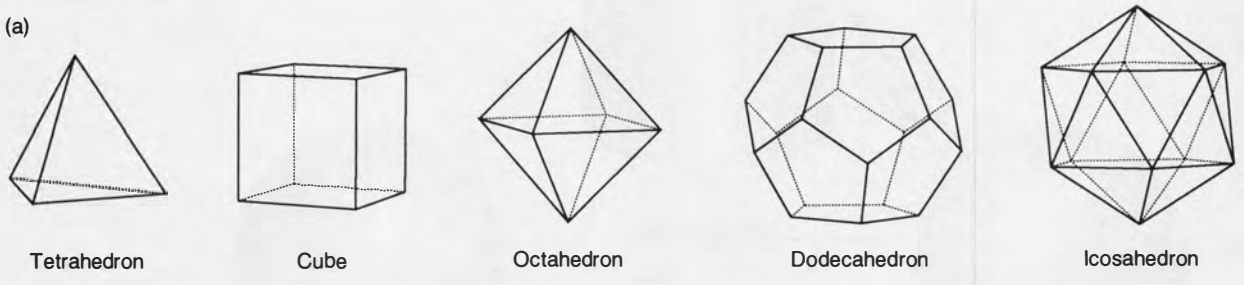
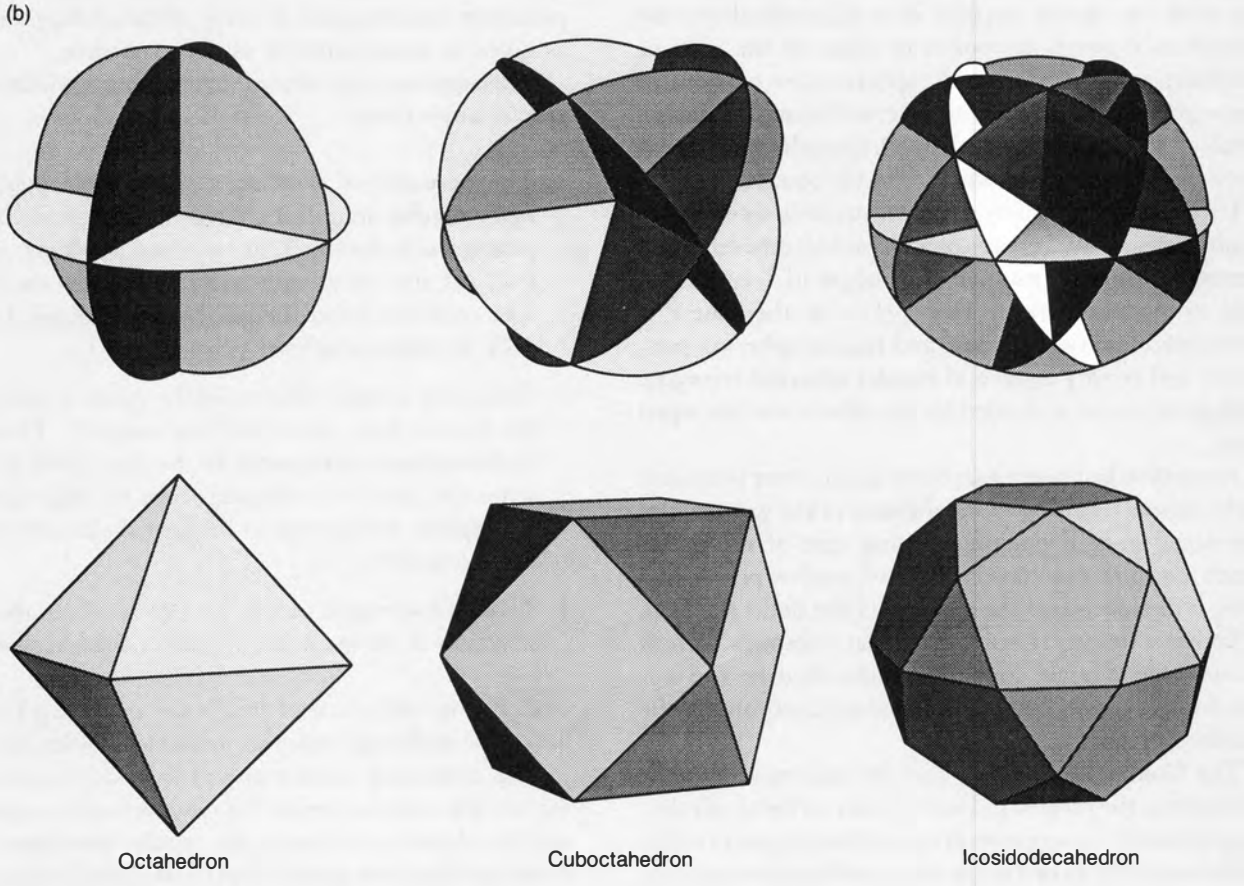


FIGURE 13. *(a) The five Platonic solids: tetrahedron, cube, octahedron, dodecahedron, and icosahedron. Each is constructed of identical faces and vertices. (b) To demonstrate the spherical derivation of the five Platonic solids, arrange hoops in the form of great circles around a sphere. The equidistant points of intersection of three hoops form the vertices of an octahedron; those of four and six hoops form, respectively, the truncated solids called the cuboctahedron and the icosidodecahedron. No more than six hoops can be arranged in this fashion. The thirty-vertex icosidodecahedron, formed by the limiting case of six hoops, contains the five Platonic solids. No other regular polyhedra can be constructed.*



cesses, on this scale, have a different *Type* of characteristic harmonic ordering. This point is later re-stated by Johannes Kepler in various locations, including his *Snowflake* paper. Modern evidence leaves no doubt of the correctness of that so-qualified observation of Pacioli, Leonardo, Kepler, *et al.*

Unfortunately, too frequently, those who point to this distinctive Platonic coherence of living processes with the Golden Section, either degrade this connection to a kind of cabalistic speculation, or simply present the Golden Section itself as a section in a circle, without showing necessity, in such popularized terms as to leave the matter of harmonic ordering vulnerable to a false charge of numerological mystification. This latter negligence appears whenever we might misdefine the Golden Section in terms of either, simply, "the Golden Mean," or as simply the derivation of the pentagon, by construction from a given circle.

If the following, restated, preconditions of rigorous treatment are satisfied, in defining the Golden Section, the risk of misleading mystification is avoided.

First, the Golden Section is located as a *necessary,* (intrinsic) metrical characteristic of negative spherical curvature, as nothing other than the characteristic distinction of the spherical generation of a subsumed, constructed dodecahedron.

Second, the five Platonic solids are recognized as each and all subsumed by the construction of a single one among them, the dodecahedron.

Third, this topic, of spherical determination of the Platonic solids' uniqueness, is referenced from the standpoint of the method we indicated above, for recognizing and solving the deep paradox inhering in Archimedean squaring of the circle. In short, that the *spherical action,* of a different, higher species than any polyhedron, bounds externally, and thus determines the constructible existence and metrical characteristics of the species of polyhedra in general.

These points are underscored by comparing the paradoxical process of squaring the circle to the way in which harmonic orderings coherent with the Golden Section *bound externally* the linear Fibonacci series (SEE Figure 14).[32] This may then be compared with Johannes Kepler's distinction between packings contrary to, respectively, negative and positive spherical curvatures (SEE Figure 15). In short, the Golden Section is a determined, necessary limit of packing of the type illustrated by the Fibonacci "growth" series under the constraint of negative curvature. With that observation, the premises for mystification evaporate.

That material covered by teacher *A,* the teacher brings the student's attention to the work of Huygens and his successors on the subjects of *tautochrone* and *brachistochrone.*[33] This leads the student through (a) *the elements of the cycloid,* (b) *the proof, by Huygens, that the cycloid is a tautochrone,* and (c) *the proof, by Johann Bernoulli* et al., *that the tautochrone is also the brachistochrone* (SEE Figure 16).

The teacher, *A,* then reviews the work which was referenced by Johann Bernoulli, Huygens' *Treatise on Light,*[34] as the next unit of study in *B*'s classroom. In this setting, *A* includes relevant references to the subject of light and hydrodynamics in the Leonardo da Vinci *Codices,* in the work of Fermat, and the treatments of a universal principle of least action by Fermat, Leibniz, and the Bernoullis. The geometrical construction employed as proofs, together with the Bernoulli experiment itself, are, combined into one, the experiment *X*; *the Bernoulli experiment itself, is the relevant physical experiment.*

This experiment shows implicitly that the universe

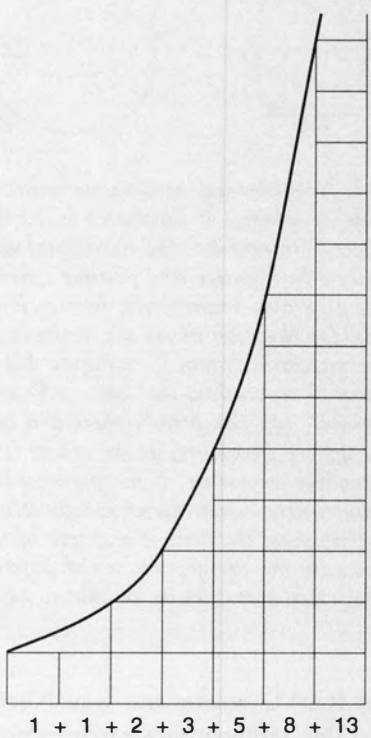


FIGURE 14. *As the circle bounds externally the 2^n-polygonal perimeter, so the linear Fibonacci series (1, 1, 2, 3, 5, 8, 13, . . . , n) is externally bounded by a curve whose harmonic orderings are coherent with the Golden Section. (Each term in the Fibonacci series is the sum of the previous two terms.) Here again, the two are of different species; paradoxically, as growth brings them into closer approximation, their species difference becomes more marked.*

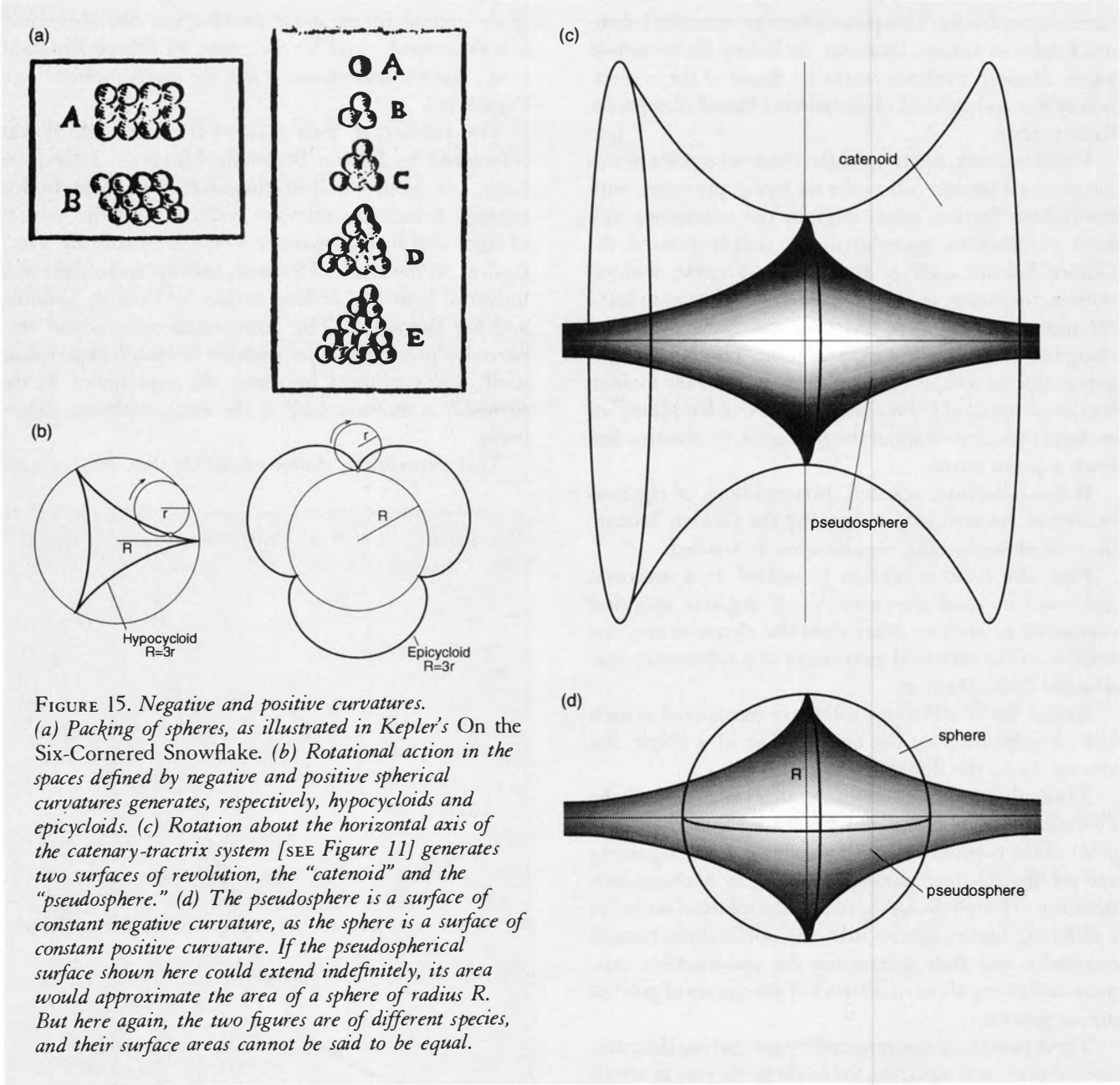


FIGURE 15. *Negative and positive curvatures. (a) Packing of spheres, as illustrated in Kepler's* On the Six-Cornered Snowflake. *(b) Rotational action in the spaces defined by negative and positive spherical curvatures generates, respectively, hypocycloids and epicycloids. (c) Rotation about the horizontal axis of the catenary-tractrix system [*SEE *Figure 11] generates two surfaces of revolution, the "catenoid" and the "pseudosphere." (d) The pseudosphere is a surface of constant negative curvature, as the sphere is a surface of constant positive curvature. If the pseudospherical surface shown here could extend indefinitely, its area would approximate the area of a sphere of radius R. But here again, the two figures are of different species, and their surface areas cannot be said to be equal.*

portrayed by René Descartes and Isaac Newton does not exist. First, the tautochrone/brachistochrone equivalence, *for the case of a constant relative speed of light,* shows that the notion of physical function in our universe requires that family of non-linear, non-algebraic functions which is derived from the isoperimetric principle. This notion of non-algebraic function supersedes all those notions of arithmetic-algebraic function derived from a notion of pairwise, linear causal interrelationship as primary. Thus the refutation of Descartes and Newton. Whereas, the non-algebraic and algebraic conceptions conflict respecting a notion of causal principle, the algebraic view is shown to be axiomatically false.

This signifies that the Cartesian domain is axiomatically false in conception from the outset. Isaac Newton's case is ultimately the same, but historically of greater ironical interest.

Newton refers to what he admits to be an absurdity of his mathematical-physics scheme, that it represents the universe as "running down," in the sense of a mechanical time-piece.[35] This "clock-winder" topic is a featured element within the Leibniz-Clark-Newton correspondence later.[36] Later, during the 1850's, Rudolf Clausius, at the prompting of Lord Kelvin, employed the assistance of

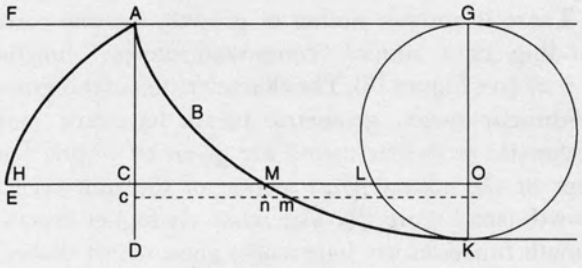


FIGURE 16. *In 1697, Johann Bernoulli solved the "brachistochrone" problem, by demonstrating that cycloid AMK, the tautochrone, was also the path of fastest descent of a body affected by gravity.*

the mathematician Herman Grassman to codify the so-called "universal entropy" dogma,[37] or "Second Law of Thermodynamics," which is nothing but a nineteenth-century version of Newton's seventeenth-century "clock-winder" fallacy. The key reference-point for discussion here, is that the seventeenth-century Newton, unlike the nineteenth-century Clausius, Kelvin, Helmholz, Rayleigh, and Boltzmann, states clearly that the fallacy of "universal entropy" erupts within his physics as a consequence of *a defect embedded within his choice of mathematics.*

This represents an important challenge for teacher *A*. *B* asks, "Does entropy exist?" "Yes," replies *A, "but not as a governing principle of the physical universe."* *B* is perplexed by this. *A* explains, by reference to Kepler, "Remember our studies of Kepler's work?"

"Remember our review of this matter in our study of Kepler's *Snowflake* paper?" Positive curvature is associated with non-living functions, such as the snowflake, which do exhibit entropy as an included characteristic. However, negative curvature requires a non-entropic ordering cohering with the limiting implications of the Golden Section.

The point here is, that, in a universe superdensely packed with spherical bubbles,[38] the envelope of all positive curvatures is a negative curvature. Thus, although some phase-states of our universe are entropic, other phase-states are not. Up to recent decades, we have known that the astrophysical realm, like living processes, is negentropic; we have found, as, for example, so-called "cold fusion" illustrates this, that the extremes of micro-space are also characteristically *negentropic.*

Thus, Newton was correct in blaming his choice of Cartesian algebraic mathematics for the "clock-winder" fallacy "hereditarily" embedded within his *Principia* as a whole.

The succession of fundamental elementary discoveries shared among persons such as *A* and *B* here, all involve significant alterations in the Socratically implicit underlying set of axiom-equivalent and postulate-equivalent assumptions.[39] The difference between Leibniz's physics, and the flawed, inferior model of Newton, helps us to recognize some leading features of that system of metaphor which is modern science practice.

Think of the elementary system of stereographic projection. Use this as an analogue of metaphor (SEE Figure 17). The sphere *NS* sits upon a flat sheet. Point *S,* touching the sheet, is termed the South Pole, and the opposite

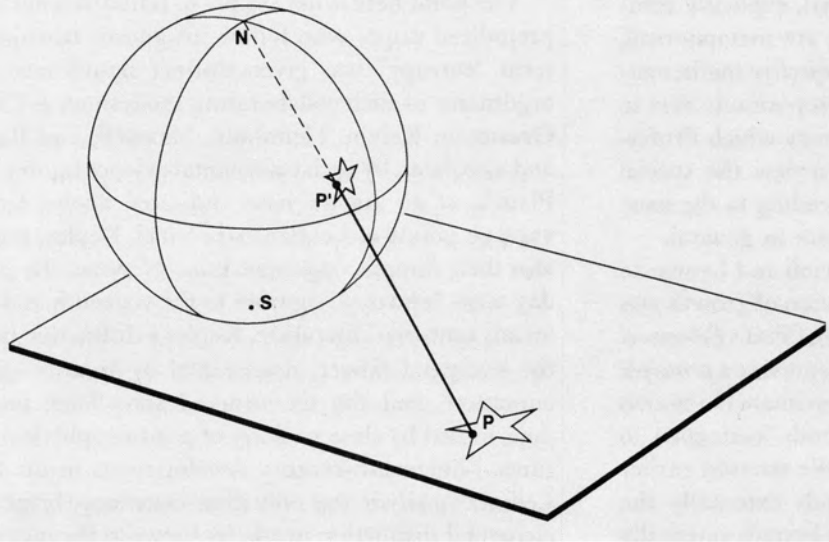


FIGURE 17. *Stereographic projection as an analogue of metaphor. Action on the sphere is projected onto the the plane. Let the image on the sphere be itself a projection from some original image in an unknown domain—that real, unseen universe hidden behind the metaphorical imageries of sense experience.*

point, *N,* the North Pole. To trace any figure drawn on the flat sheet onto the surface of the sphere *NS,* draw a moving ray from the North Pole to the sphere to the outline of the figure on the flat sheet. Where the moving ray cuts through the surface of the sphere, there lies the spherical image of the relevant trace figure upon the flat sheet.

Now reverse the projection, from a figure on the sphere, to a shadow cast by the moving, tracing ray upon the flat sheet. Then, add a third feature to this; that the image on the sphere itself be a projection of some original image in an unknown domain, that real, unseen universe, hidden behind the metaphorical imageries of our sense experience. Let this unseen, real universe be approximated, *metaphorically,* by the Cantorian *Type* of the *aleph*-manifold as a whole. Let the domain of scientific physical functions, in the mind, be represented, metaphorically, in communication, by the analysis-situs-enriched, extended *Type* of non-algebraic functions in general. Then, thirdly, let the lowest order, the linear world of Aristotelian nominalist sense-certainty, be represented, metaphorically, by the *Type* of systems of deductive theorem-lattices.

Those three levels, combined so, represent, metaphorically, the domain of metaphor. The notion of a *Type* which subsumes all the possible relationships among these three, including matters of physical science, but also classical forms of drama, poetry, and music, we define here as the function of metaphor.

Negentropy

Before passing beyond the thought-objects of physical science, to the classical art-forms, we have two final matters to settle respecting physical science. One of these two is, obviously, the query, "If formal, explicitly communicable aspects of physical science are metaphorical, what happens, then, to the idea of an *objective* mathematical physics?" The other of the two propositions next to be considered is that notion of *negentropy* which Professor Wiener so crudely abused. We review the special topic of negentropy first, before proceeding to the issue of objectivity of formal physical science in general.

Prior to the referenced work of Pacioli and Leonardo da Vinci, the mathematical representation of growth was given, as we noted above, by Leonardo of Pisa's *Fibonacci Series.* The Fibonacci series does not represent a *principle of growth,* but only an attempt to approximate the notion of growth descriptively, using methods analogous to Archimedes' squaring of the circle. We stressed earlier here, that the Golden Section bounds externally the extended Fibonacci series, as the circle bounds externally the 2^n-polygonal perimeter; that which bounds, is of a different, higher species than that which is bounded; the higher cannot be derived "hereditarily" from the lower.

There is another notion of growth, the one corresponding to a simple "compound-interest" function, $(1 + x)^t$ (SEE Figure 18). The characteristics of this growth (arithmetic mean, geometric mean, harmonic mean, arithmetic-geometric mean) are given by elliptic functions of the conical cross-section of the unit cycle of growth (SEE Figure 19). The relatively higher orders of growth functions are hyperconic ones, which shows us that generation of increasing density of apparent discontinuities which is the observable characteristic of growth *per se,* or *negative entropy (negentropy).*

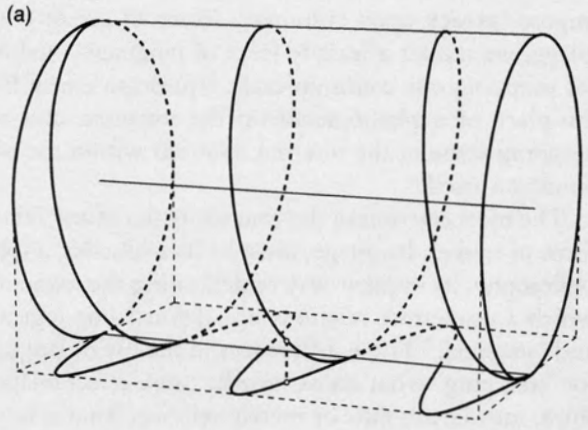
In other words, it is not possible to represent *growth* of this *Type* characteristic of living processes by means of a deductive form of mathematics, such as a Fibonacci series, the mathematics of pairwise interactions along straight-line pathways. The attempt to define negentropy, as Wiener does, by means of Ludwig Boltzmann's statistical mechanics ("H-theorem"), is simply outright incompetence from the outset of such an endeavor.[40]

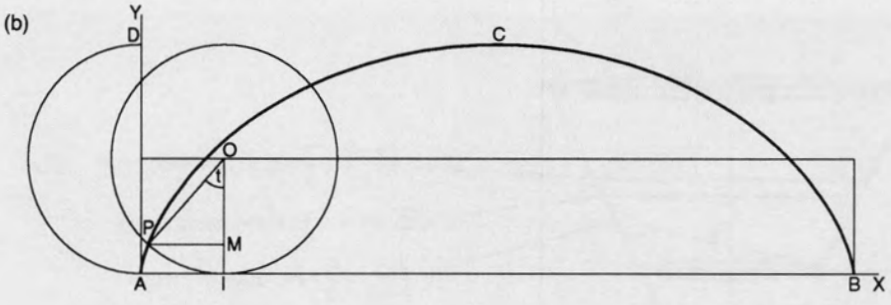
The characteristics of human scientific progress are, as we have indicated earlier, changes in the axiomatic basis of theorem-lattices which are of a *Type* of the Cantorian *aleph*-manifold. Biological evolution is a process of this same formal *Type*. Kepler's universe is, ultimately, of this same *Type*. Until the twentieth century, we observed this *Type* of process in living processes, in manifest results of creative mental discoveries, and in Kepler's implicit ordering of the universe as a whole. Recently, we observe the same underlying pattern of elementary causation as we approach phenomena of physical chemistry on the scale of 10^{-16} to 10^{-17} centimeters.

The point here is not simply to refute Wiener and the prejudiced dupes who follow his gnostic teaching. The term "entropy" was given distinct significance by the arguments of such collaborating spokesmen as Clausius, Grassman, Kelvin, Helmholz, Maxwell, and Rayleigh, and also, later, by such continental-science figures as Max Planck, *et al.* As we have indicated above, from the vantage-points of Leonardo da Vinci, Kepler, *et al.,* and also their famous opponent Isaac Newton, the present-day term "entropy" signifies to the sixteenth and seventeenth centuries' literature, Kepler's distinction between the five-petal flower, determined by *negative* spherical curvature, and the six-cornered snowflake, the latter determined by close packing of positive spherical curvatures. Nineteenth-century developments in the field of Leibniz's *analysis situs* only illuminate more brightly that elemental distinction in species between the intrinsically

entropic "hereditary" characteristic of positive curvature, and the intrinsically *negentropic* characteristic of negative spherical curvature.

It should be evident, on these very elementary grounds, that the mathematical schema of Grassman, upon which Clausius and Kelvin's introduction of the so-called "Second Law of Thermodynamics" depends, is a fraud, a fallacy of composition akin to recognizing only one side of the set of terms of Schrödinger's ψ-function.[41] Since the work of Pacioli and Leonardo, or, since that





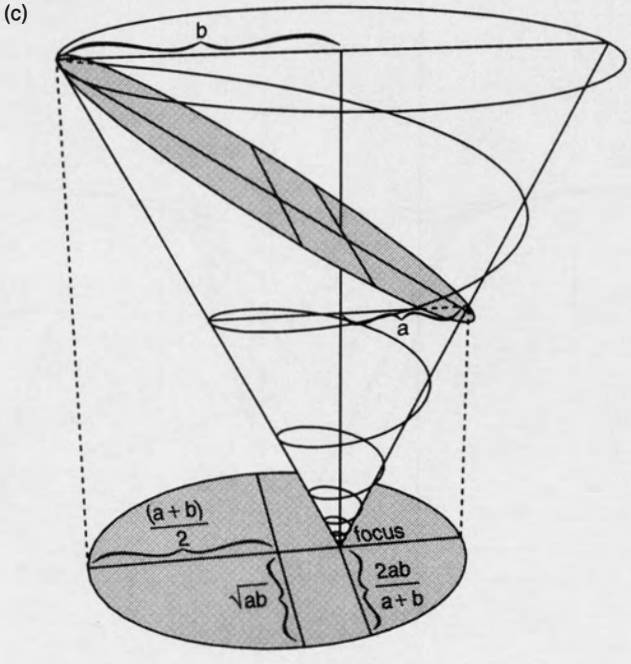


FIGURE 18. *Simple growth functions. (a) Rotation up a cylinder, when projected onto a plane, generates a sine wave. (b) The sine wave is a co-function of the cycloid. Given cycloid ACB, using rectangular axes ABX,AY, the coordinates (x,y) of any point P on ACB are given by*

$x = at - a\sin t = a(t - \sin t)$,

$y = a - a\cos t = a(1 - \cos t)$.

(c) Rotation up a cone (spiral action), when projected onto a plane, generates elliptical functions, which are coherent with the characteristics of simple growth functions—for example, with the intervals of the well-tempered musical scale.

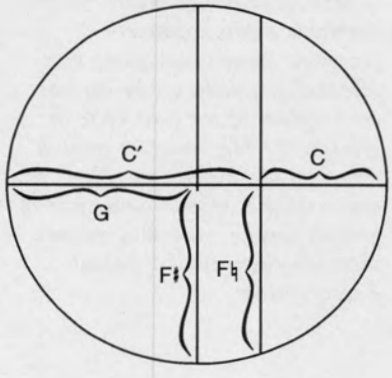
a is the radius at perihelion

b is the radius at aphelion

$2ab/(a + b)$ is the harmonic mean, which occurs at the latus rectum

$(a + b)/2$ is the semi-major axis

$\sqrt{ab}$ is the semi-minor axis

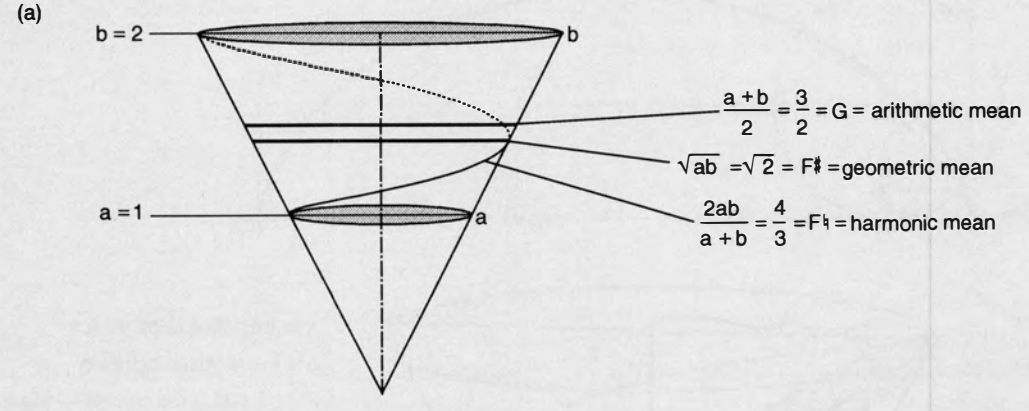


of Kepler, positive spherical curvature bounds externally a system of linear inequalities (functions) which are pervasively, "hereditarily," characteristically *entropic;* but, negative spherical curvature, the externally bounding curvature of universal physical processes, generates processes which are, like life itself, characteristically *negentropic.*

These "hereditary" distinctions in harmonic orderings, between positive and negative modes of spherical curvature, obviously pertain to the metaphorical domain of extended non-algebraic function, the which is the middle, second of the three levels of a function of metaphor. This apparent negentropy, of negative spherical-curvature harmonics, is, of course, externally bounded, subsumed by the higher Cantorian *Type* associated with the *aleph*-manifold. Negentropy does not exist, as a concept of a governing principal process on the level of any single deductive theorem-lattice.[42]

There is plainly no intrinsic error in employing commonly accepted names as pointers for indicating the respectively appropriate kinds of phenomenon. Absurdity, veering toward insanity, enters if we tolerate the radical nominalist proposal, to base our belief respecting the intrinsic physics of phenomenon upon the dictionary definition of those mere terms. However, communication is not limited to pointing while uttering a noun or nominative phrase. In the civilized efforts to impose literacy upon customary forms of use of a language, we render a literate form of language a method for mapping our communicable representation of both the place of a phenomenon in the universe, and also mapping some of the internal relations within the phenomenon itself.

The most relevant of the characteristics of any literate form of spoken language, must be its adducible implied philosophy, its implicit way of delimiting the manner in which cause-effect relations are defined ontologically, and "mapped." These differences in the use of language for "mapping" what are assumedly cause-effect relationships, may be absolute or merely relative. That is to say,



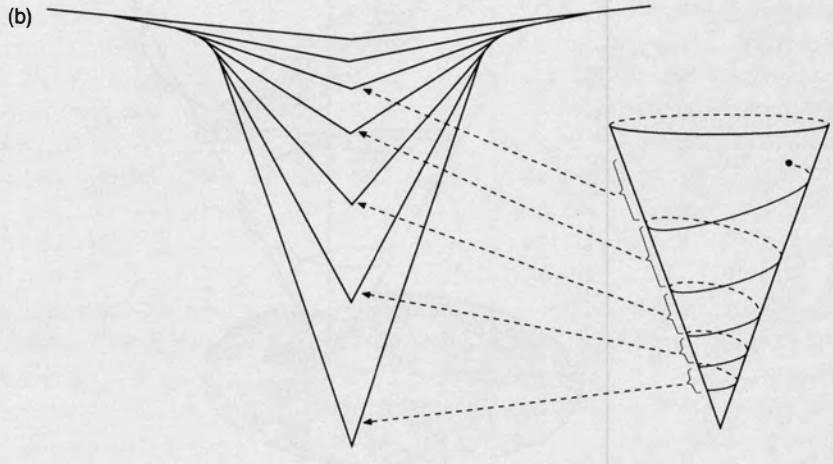


FIGURE 19. *Simple growth and negentropic growth. (a) The characteristics of simple growth—arithmetic mean, geometric mean, harmonic mean, arithmetic-geometric mean—are given by elliptical functions of the conical cross-section of the unit cycle of growth. (b) Higher-order growth functions are hyperconic. Here, the characteristics of each unit cycle of growth develop according to the increasing density of apparent discontinuities.*

they are *absolute* if they inhere in the accepted forms of use of that language; they are *relative,* if they reflect one among several optional forms of use of that language in currency.

Consider the similarity to "mathematical languages."

There are absolute philosophical differences separating the reductionist algebra of a Descartes from the non-algebraic representations of function of a Leibniz. Yet, as long as we limit the use of reductionist algebra to mere description of ordinary kinds of non-crucial phenomena, algebra can be a useful tool. One must recognize there are circumstances under which the intrinsically inferior, philosophically false method of such an algebra must be avoided, and the superior, non-algebraic method is mandatory—in treating topics bearing upon least action, for example.

This consideration brings us to a higher degree of metaphor.

At the beginning here, we have emphasized the simplest aspect of our topic, the metaphorical relationship between a single term and an unutterable, but real, individual thought-object. Now we have to consider a higher order of thought-object; we must consider the point, that entire statements, statements which purport to "map" cause-effect relationships, even entire books sometimes, may also be metaphors for single thought-objects. Turn, now, to an elementary illustration of this point.

In our treatments of some elementary thought-objects of scientific work, thus far, we have considered some crucial thought-objects originally attributed to such authors as Pythagoras, Plato, Archimedes, Euclid, Nicolaus of Cusa, Luca Pacioli, Leonardo da Vinci, Johannes Kepler, Pierre Fermat, Christiaan Huygens, Gottfried Leibniz, and Johann Bernoulli. Associate the original form of that true paradox and its solution as associated with a name, a portrait, and a brief biographical sketch of that author. Those images you now associate with a corresponding memory of your re-experiencing the production of the relevant thought-object originally experienced by them.

How should one order the seating of these discoverers in one's memory? For scientific work, the primary consideration must be, not raw chronology as such, but, rather, the rather obvious principle of "this necessary predecessor" among crucial discoveries as a whole. That ordering principle permits a range of equally valid, but different orderings among the same array of discoverers. Each of those choices of orderings among arrayed thought-objects, is a distinct thought-object, with the included quality of a Cantorian *Type*, indeed, subsumed by the *Type* of an *aleph*-manifold.

Consider an obvious choice of illustration here. We have begun with Nicolaus of Cusa's 1430's discovery of an isoperimetric principle paradoxically underlying theorems of Archimedes. That isoperimetric notion, as elaborated by Cusa,[43] set the stage for a range of crucial discoveries by Pacioli, Leonardo da Vinci, *et al.,* at the close of the fifteenth century and the first decades of the sixteenth. The treatment of Plato's discovery of the Golden Section's implications, by Leonardo *et al.,* referenced Plato, Archimedes, Euclid, and Cusa (and, probably also the *De Musica* of St. Augustine).[44] This, in turn, set the stage for the most crucial of the discoveries by Kepler. Leonardo da Vinci, on the same basis, developed the crucial discovery of the transverse wave-function for electromagnetic propagation, and the finite speed of light later first measured (approximately) early during the seventeenth century. The work of Leonardo da Vinci, Kepler, Fermat, Desargues, and Pascal, informed the discoveries of Huygens, Leibniz, and the Bernoullis on least-action principles and the related features of non-algebraic functions.

If we examine the arrangement by the rule, that the passage from one or more crucial discoveries to a successor crucial discovery, must always occur in the paradoxical manner we generate, initially, an individual thought-object, the result of constructing this choice of order by that method, is to generate a higher-order thought-object, of that quality which subsumes the successive generation of the constituent thought-objects in that selected array. This higher quality of thought-object is therefore of a distinct Cantorian *Type*: that array of thought-objects, considered as they might have been generated in that selection and assigned order, generated by a constant principle of difference, forms a manifold, or sub-manifold of this description. The higher quality of thought-object generated by that ordered array is the *Type* of that manifold or sub-manifold.

From that standpoint, all communication relating to significant ideas is necessarily, intrinsically metaphorical.

The progressive ordering of a succession of thought-objects in this way, according to such a higher quality of thought-object *Type*, is the phenomenon which corresponds to what we ought to signify by the term *negentropy.* For classroom purposes, we signify the case in which a series of successively higher states of organization is generated according to a principle which intrinsically orders such a succession.

For example, let us imagine that at a certain point the higher state of matter in our universe is either a population of neutrons or hydrogen atoms. By combining these, through fusion, a periodic table of elements and their isotopes is generated. Where lies the negen-

tropy in this image of fusion development? Is it that lithium might be a "higher state of organization" than hydrogen? Or, is it not that the universe is now populated by neutrons, hydrogen atoms, and also helium and lithium atoms? The point being illustrated, is that higher states of organization of the process *as a whole* are being generated successively, in accord with a higher, subsuming principle of a periodic table in general: in that latter aspect of the phenomenon lies the true negentropy.

The Essential Subjectivity of Science

Lurking among the numerous accomplishments of modern science, there is the absurd, but popular delusion, that "physical science" is both "materialist" and "objective." The worst, and most widespread forms of this delusion assume, first, that scientific method is essentially statistical, and that "mathematical science" is associated with measurement of forces acting along a straight-line pathway between two points. This popular delusion was key to the widespread "systems analysis" hoaxes, such as Professor Norbert Wiener's "information theory."

The proof, that such definitions of "objective science" are absurd, is elementary; that proof is given as a central feature of this author's introductory course in Leibniz's science of Physical Economy.[45] We summarize the background considerations, point by point.

1. If man were a mere animal, that is, like a baboon, a creature innately disposed to what is called "primitive hunting and gathering" modes of social reproduction, at no time could the living human population of this planet have exceeded about ten millions individuals.
2. The increase in the human population, and the associated improvements in life-expectancy and standard of existence, are the cumulative benefit of what we may identify most simply and fairly as "scientific and technological progress." The measure of this function of progress is an increase in the potential population-density of the human species; this represents a higher *per-capita* standard of living and longevity, combined with a decrease in the total number of hectares required to sustain an average individual human life.
3. These improvements are expressed functionally through a succession of successful, radical changes in human productive behavior, a succession akin to the series of discontinuities associated with *A, B, C, D, E,* . . . referenced above. These changes in the behavior of successive levels of upward development of society are analogous in form or function, and effect, to successful, upward biological evolution of species among the lower forms of life.[46]
4. Thus, the problem of both discovering and choosing a *Type* of sequential ordering of thought-objects, corresponding to a negentropically ordered succession of revolutionary scientific modifications in known scientific principles, is a *subjective* matter. It is a matter of discovering which subjective *Type* of creative-mental generation of thought-objects corresponds to a negentropic sequence of increase in man's cultural *potential for increasing potential population-density.*

Thus, from this point of view, the subject of *science* is that higher-order of thought-object—a *transfinite*—which correlates formal scientific progress with rate of increase of this science-driven rate of growth of a culture's potential population-density. In other words, man willfully increasing mankind's power to perpetuate evermore successfully his own species' dominating existence within the universe.

This view is in contrast to the popularized materialist *mythos* of so-called "objective science," of man as the contemplative mathematician-observer.

"*I see myself creating,* as I define creation, as a common principle of that array of named thought-objects of fundamental discovery associated with such as Plato, Archimedes, Cusa, Leonardo, Kepler, and Leibniz. I locate my own creating-activity in respect to an effort to attribute a higher thought-object, a Cantorian *Type,* to the manifold composed of such historic names of original discoverers. This attribution of a specific choice of order for such an '*aleph*-manifold,' and of attributing a *Type* to that choice of ordering, is the immediate subject of my inquiry.

"This *Type* defines a relatively fundamental scientific principle, as an hypothetical choice of such a principle; in Plato, this is referenced as 'hypothesizing the higher hypothesis.' I now correlate that hypothetical choice of *Type* with a manifest ordering of science-driven growth of relative potential population-density, of relatively superior and inferior modes of physical-economic culture."

This correlation is the characteristic activity of physical science; seeking to subsume all such hypothesizing of the higher hypothesis as a manifold of a yet higher *Type,* is *physical science.*[47]

As described in other locations,[48] the details of this phenomenon are of the following form. The hypothetical inference of a new *Type* of ordering of crucial thought-objects of fundamental scientific discovery as a manifold (or, sub-manifold), in respect to a single *Type* of crucial (or, "unique") paradox, subsumes an experimental design for some crucial expression of this new hypothesis. That subsumes, in turn, the design of either an experimental apparatus, or an observational method akin to such an apparatus.

Thus, from fundamental discovery of (transfinite) ordering-principle, through the design of an experiment, through that experimental design expressed as a new principle of machine-tool (or, analogous) design, is the generation of a discovery of scientific principle transmitted and assimilated into a general increase of social productivity. In every step of that process, the essential thing is the generation of a new conceptual thought-object by, within, and in accord with the sovereign, individual creative mental processes of the mind of a sovereignly individual person.

We should emphasize by aid of such means as reiteration, that the process just outlined is Plato's "hypothesizing the higher hypothesis." The higher hypothesis is the *Type* of cardinality to which corresponds a manifold (or, sub-manifold) of thought-objects arranged in a certain choice of ordering. The choosing of such a particular such higher hypothesis, the hypothesizing of the selection of one or more such higher hypotheses for such an array of individual thought-objects, is itself the consideration of a manifold of such alternative *Types.* The latter manifold's *Type* is what we should signify by *physical science.*

In other words, physical science is essentially the process of discovering those rules of creative behavior of our individual mental processes which lead us to discoveries of a *Type* through which general culture may be changed to optimize the rate of increase of our species' potential population-density. In this fashion, physical science is essentially subjective.

Admittedly, that does not complete the argument. If a certain type of "hypothesizing the higher hypothesis" is physical science, then increases in potential population-density, so successively achieved, show us that the intelligible form of lawful ordering of nature is coherent with the process of perfection of our hypothesizing the higher hypothesis. Thus, it is our successful hypothesizing of the higher hypothesis, in this fashion, rather than our sensory impressions, the which is the proper basis for determining the lawful composition, and ontological characteristics, of that real physical universe which lies beyond the full reach of our mere senses.

Our creative-mental processes do not address directly sensory objects as sensory objects *per se*. Human thought knows only *change;* we know only a thinkable correspondence between a change in our behavior and a correlated change in the manifest behavior of nature. It is a correspondence of the two *Types* of change which constitute the entirety of real physical science. That correspondence is what is intelligible for us; we must discover everything else respecting nature from this approach to the elementary primacy of change, to the universal elementarity in space-time of nothing but change.

This point is clearer, if we look now at the historical source of the leading opposition to the picture we have presented.

The 'Materialist' Opposition

The leading opponent of our Leibnizian view of science, and the modern opponent of Plato, Cusa, Leonardo da Vinci, Kepler, and Leibniz, for example, is the so-called "materialist," or "mechanistic" standpoint of Francis Bacon, Robert Fludd, Elias Ashmole, René Descartes, John Locke, and Isaac Newton. This "materialist" dogma was introduced to seventeenth-century France and England by the then newly-established cult of the Rosicrucians. The essence of this gnostic Rosicrucian dogma is typified by René Descartes' *deus ex machina*[49] and Isaac Newton's maxim *hypotheses non fingo.*[50] This is also the axiomatically "hereditary" origin of such modern forms of radical positivism as von Neumann's "systems analysis," Professor Noam Chomsky's Korschite "linguistics," and Wiener's "information theory" hoax.

Consider as much of this Rosicrucian cult's dogma as is essential to locate the origins of that popular delusion we recognize most readily as the mythos of "objective science." The derivation of the Rosicrucian cult is the best vantage-point for this undertaking.

The seventeenth-century Rosicrucian cult of Fludd, Ashmole, *et al.* was a resurfacing of a notorious, usury-practicing, medieval sect known variously as the *Cathars, Bogomils,* or, more commonly, "The Buggers."[51] This sect, which infested the market centers of northern Italy and southern France ("Languedoc"), was one of many varieties of kindred gnostic cults sprung up over the centuries from such very ancient pagan origins as the Phrygian cult of Cybele-Dionysus, the Delphic cult of Apollo-Dionysus, the Hellenic cult of Osiris, and the sundry Babylonian and Canaanite mystery religions.

The relevant feature of these gnostic forerunners of Ashmolean Rosicrucianism is the doctrine of utter depravity of the "flesh" which is the direct source of the materialist dogmas of Bacon, Descartes, Locke, Newton, *et al.* The sexual perversions of the Cathars are a direct, doctrinaire correlative of this materialist dogma of theirs. Briefly, *one of the cult's Elect was forbidden to place his semen in the vagina of a woman, lest he cause the procreation of newborn human flesh! The spirit inhabiting the Elect must be kept apart from the utter depravity of the fleshly process of human procreation.*[52]

That said, consider the case of science-driven increase of society's potential population-density. The origin of a new, valid, fundamental discovery, is a mental act of creation, *a spiritual act,* the generation of such a thought-object. The derivation of a design of experimental apparatus, and then a machine-tool principle, from the new

thought-object, is the source of a powerful *material effect.* This is the connection which the Rosicrucian Descartes insisted must be broken: *deus ex machina,* and which Newton forbade: *hypotheses non fingo.*

What kind of society do these Manichean, or Bugger Elect represent? The Elect are forbidden to interfere with nature; they cannot till the soil, nor perform other productive labor. They are permitted to subsist by begging for alms, or to loan their accumulation of monetary savings from alms-gathering in *usury.* The Elect form, thus, a parasitical class subsisting by tribute and usury.

The strength of such a usury-practicing gnostic conspiracy, is that the Elect of the "Bugger" sect could sell a note for twelve or more ducats in Lyons, which could be redeemed by the bearer at discount for ten ducats, or less, in Padua. Thus, spider-web networks of Elect "Buggers" spread across northern Italy and southern France of the Garonne-Tarn and Rhône regions, in symbiosis with the other principal usury-practicing "Elects" among Lombard bankers and Jewish moneylenders.

The following summary is fair. As the oligarchical, usury-practicing *I Nuovi* faction of the Venetian merchant-bankers spread their parasitical, oligarchical power, by such vehicles as the Levant Company, into England, the Netherlands, and the old Hanse regions of Northern Europe generally, the Netherlands and England became the target for the launching of such Levant Company spin-offs as the Bank of England, the City of London financial center, and the Dutch and British "India" companies. London became thus the "new Venice," a union of the usurious Levant Company "Lombards" with the Rosicrucian cult of Bacon, Ashmole, *et al.* These seventeenth-century developments were the roots of the combined work of the Liberal Party and (later) Fabians of the eighteenth and nineteenth centuries, in seeking to establish London as the capital of a "Third Roman Empire," a worldwide form of *pax universalis,* a British Empire which would be a revival of the pagan Roman Empire of Augustus, Tiberius, Caligula, Nero, and Diocletian.

Originally, science was solely a creation of the Platonists of the Golden Renaissance, chiefly the work of those fifteenth-century moral and intellectual giants who are best typified by Cardinal Nicolaus of Cusa and Leonardo da Vinci. This tradition was continued by the work of such as Kepler, Gilbert, Fermat, Desargues, Pascal, Huygens, Leibniz, and the Bernoullis. That seventeenth-century Leibnizian tradition was carried into the nineteenth century by such figures as France's Gaspard Monge, and Germany's Gauss and Riemann. This tradition is sometimes called "continental science," to distinguish it from the Cartesian, empiricist, and positivist outgrowths of the Rosicrucian influence.

The cases of Bacon, Fludd, Descartes, and Newton established the counter-science variously expressed as *Cartesianism,* empiricism, and *positivism.* The hegemony of this cult's "Enlightenment" materialism in most science classrooms today, is the result of British participation in victories in most of the wars of the past three hundred years. The supremacy of the Rosicrucian's materialist dogma in today's scientific establishment is not a scientific, but a purely political phenomenon.

The practical issue of this political division in the science establishment, is the overarching conflict between the two principal, conflicting social systems which have, almost entirely, dominated European history since Solon's defeat of the oligarchical usurers of ancient Athens, more than two-and-a-half thousand years ago. This is the point made by Friedrich Schiller's contrasting the humanist, republican constitution of Solon to the American-Confederacy-like law of Lycurgus' Spartan slave society.[53]

To sustain scientific and technological progress requires appropriate education of virtually all participants in the society's productive processes. A population so educated will not tolerate indefinitely that division of society's population into oligarchs and helots which was characteristic of Lycurgus' Sparta, the pagan Roman Empire, and the American Southern Scottish Rite Jurisdiction's Confederate States of America. The brutish ignorance to which the slaveholders' oligarchical system degraded not only the Confederates' "poor whites," but also most of the so-called "planter aristocracy,"[54] illustrates the point at issue. The so-called "socialist" zero-technological growth decrees of the Roman Emperor Diocletian are a notable, consistent precedent for the brutish degeneracy pervading the old Confederacy.[55]

On the other side of the same issue of policy, an ignorant people is not capable of self-government. To govern oneself requires the capacity for efficient comprehension of qualities of processes which are, by their nature, intrinsically beyond the developmental capacity of the scientifically illiterate strata. As several founders of the U.S. federal republic warned, the survival of such a democratic republic as theirs under natural law required a certain minimal quality of compulsory education.[56] Friedrich Schiller presented the conceptual basis for the most successful model of Christian classical humanist education, the reforms of Wilhelm von Humboldt.[57]

Under the influence of such a quality of universal compulsory secondary education, that educated citizenry will conspire to free itself from any oligarchical rule.

Yet, without such an intrinsically anti-oligarchical form of education, a society could not generate, transmit, or assimilate efficiently scientific and technological progress in a general way. The self-interest of the oligarchy, as a social formation, is to destroy nations practicing generalized scientific and technological progress, and then seek to outlaw, throughout the world, both classical education and the practice of scientific progress. That is the *entropic Type* of cultural policy represented by the "(guild) socialism" of Diocletian, wherever the like appears, down through the ages of history since not later than the Phrygian Cybeline cult of Dionysus.

Like Kant's pro-irrationalist *Critiques* later, Descartes' gnostic *deus ex machina* dogma sought to paint a picture of the material world independent of that indispensable subjective agency, the creative mental processes upon which the discovery of all scientific knowledge depends absolutely. Kant did not deny the efficient existence of creative powers of scientific discovery, but pronounced deliberative creative acts to be impossible.[58]

That is the kernel of what passes for sophisticated philosophical materialism. To the credulous simpleton, the materialist demagogue exhibits himself as a solid, down-to-earth good fellow, one, perhaps, with all four feet firmly planted on the ground. "We materialists believe in nothing we cannot experience first-hand, with our own good five senses." To thoughtful, literate audiences, such cheap rhetoric is not persuasive; the argument of the Kantian unknowable *thing-in-itself* and Descartes' *deus ex machina* is offered, instead.

For us, the relevant experience on which physical science must be premised, is not fixedness, but *change:* the correlation of a change in our scientific thinking for practice, with the resulting change in the responsive behavior of nature. Unlike that theology as such which references the *Absolute* of Plato's the *Good,*[59] mere physical science does not know the *Absolute,* but only Cantor's *Transfinite.* The domain of the transfinite is, at its highest level, Plato's *hypothesizing the higher hypothesis,* the domain of physical space-time, the domain of *change,* of perfecting that which remains unperfected. Thus, for physical science, the science of physical space-time, experience is *change,* and change is the *elementary substantial* feature of all scientific experience.

As the illustrative case of the experiment shows, *change* begins as an ostensibly non-material, subjective act of valid creative discovery of new, un-utterable *Geistesmassen—thought-objects.* This first step in the causal sequence of human action is *spiritual,* not "material." Under the "foremanship" of the relevant thought-object, a crucial experimental design is fashioned, a material medium for the *spiritual cause,* which latter is the thought-object. So, we had next, the derivation of the new machine-tool principle, and the medium through which man's *per-capita* power over the universe, per square-kilometer, is increased. The latter is the relevant *material effect.*

It is this sequence, this spiritual change causing the material change, which every successful experiment demonstrates. The materialist insists that the results of the experiment must be described only in such ways as leave the generation of the relevant new thought-object out of account. Since the universe responds to the experiment as it is actually developed, as prompted by an initially spiritual cause, materialism, with its materialist's fanatic adherence to formal deductive *consistency,* falsifies the universe by such reductionist fallacy of composition.

II. Metaphor As Classical Tragedy

During 1948-1952, the period this author first completed the theses presented afresh here, he thought that to prove his case, that Wiener's "information theory" is a dangerous hoax, one had to direct against *radical positivist* Wiener the same form of refutation which this author had then earlier composed against the elementary fallacies of Immanuel Kant's virulently anti-Leibniz *Critiques*;[60] the last of those, Kant's *Critique of Judgment*[61] may be taken as a point for our purposes here.

This meant, then as now, that one must first attack Kant's neo-Aristotelian formalism, Kant's formal, reductionist pseudo-proof, that creative processes of original scientific discovery of principle are unknowable *a priori.* Additionally, it was clear then, as now, that just as Kant goes from this, in his *Critique of Judgment,* to deny any rational principle of knowledge of aesthetics, so we must show, that the same intelligible principle underlying creative, valid, original scientific discovery of principle, must be the governing principle of creativity in classical fine arts.

Then, in 1948-1952, as now, our central focus was that step-wise relationship between crucial scientific discovery and employment of derived machine-tool-design principles we have identified in this present location earlier. To show this same principle at work in classical fine arts, we focused upon classical poetry, emphasizing the chosen cases of Schiller and Goethe, and, then, using Goethe as the vehicle for treating the German *Lied* as represented by Mozart, Beethoven, Schubert, Brahms,

and Wolf settings of Goethe.[62] Here, we emphasize, instead of the *Lied,* the model of classical tragedy.

The disadvantage of employing classical tragedy as an illustration, is that there are so few truly notable tragedians, as distinct from great classical composers (from Praetorius through Brahms). Only Aeschylos, Cervantes,[63] Marlowe, Shakespeare, and Schiller, chiefly, are exemplary of truly successful tragedians. Only the historian Schiller, among these few, mastered explicitly a statement and demonstration of the principles of composing classical tragedy. Nevertheless, the compelling advantage of using the case of tragedy here, is that, implicitly, it most perfectly situates in art-form the Cantor notions of *cardinality* and *power* (the German *Mächtigkeit*), as Cantor defines these to include the problems of ordering the *aleph*-manifold.[64]

Consider as classical drama the array of exemplary crucial scientific discoverers we listed here earlier: Pythagoras, Plato, Archimedes, Cusa, Leonardo da Vinci, Kepler, Gilbert, Desargues, Fermat, Pascal, Huygens, Leibniz, the Bernoullis, Gaspard Monge, Karl Gauss, Bernhard Riemann, Eugenio Beltrami, and Georg Cantor. Arrange the crucial discoveries associated with these personalities, to imply an ordering-principle (higher *equivalence, Type*) which we may equate metaphorically to the name of *science.* Then, construct a contrasting, *entropic* array, typified by such followers of Rosicrucian "Buggery's" materialist principle as Bacon, Fludd, Hobbes, Descartes, Locke, Newton, Cauchy, Clausius, Klein, Kronecker, Helmholz, Maxwell, Rayleigh, Boltzmann, Russell, von Neumann, Wiener, *et al.* This is an *entropic Type* which we may rightly equate metaphorically to the name of *anti-science.* There, we have the principal historical background elements of *dramatis personae* from which to conduct a truly classical tragedy according to Schiller's principle.

The basis for constructing a drama inclusive of these two, mutually exclusive *Types*, is that the formal elements of each of the manifolds might each reference the same phenomena of scientific history as the other, although the ordering principle by means of which the opposing *Type* knows the element *metaphorically* may be totally irreconcilable with the opposing one.

The tragedy based upon such a conjunction might be built up in the following way.

Given, a society whose prevailing custom in science is the "post-modernist" version of the *entropic Type,* but a society in which a few potential heroes know that the crucial elements of the society's scientific-economic practice might be ordered according to the *negentropic Type,* as readily as to the presently hegemonic entropic choice. Define a situation in which the failure of a potential hero to act with pungency and force upon that latter option, means a devastating military or other kind of great suffering for his or her nation. Let this unhappy consequence occur, ostensibly because the potential hero fails to seize his last available opportunity, at the *punctum saliens,* to bring about the required shift of emphasis in the society's policy practice. The potential hero's seizure by what is fairly termed "psychosexual impotence," like Hamlet's, takes the form perhaps, of fearing to lose his academic security or pension, should he confront directly the entropic faction in this matter.

This failure of the potential hero defines the tragedy. This failure is implicitly of an intelligible Cantor *Type*; but, that is not a fully adequate representation of the notion of this tragedy.

The tragedy is performed before an audience. The performance of that drama, presented to that audience, begins to succeed if the audience is made conscious of the opposition of the two *Types*, and of the potential hero's situation. Thus, the audience, by taking the dramatic character's express consciousness as the object of the spectator's conscious attention, is seeing the drama, and the characters depicted, as if from above. If the audience also recognized something of itself in each of these characters, the drama has reached a second milestone in the direction of success.

Next, the negentropic alternative must ultimately uplift the spirits of the spectators; that is the spark of true life, evoked so within the audience, and imparted thus, by fusion, to the audience's consciousness of the succession of dramatic montage sensed of events on stage.

This assembles a manifold of not less than the following conjoined arrays of thought-objects: (1) the *negentropic Type* of science, as portrayed; (2) the *entropic Type* of anti-science; (3) the manifold of the phenomena which both of the foregoing two, opposing *Types* reference as correlation of their respective thought-objects; (4) the role of the potential hero, as a thought-object; (5) the audience's reflection of its own projected consciousness of itself, returned to it as fused with the drama on stage.

This manifold, so composed, must be represented by a thought-object corresponding to the tragedy as a whole. That *Type* is the indivisible substance, the sovereign elementarity of that tragedy as a creative work of classical fine art. Here, metaphor is plainly the indispensably ironic character of both every moment of development of the drama, and of the drama in its entirety.

The idea of metaphor as mere "symbolism," is plainly the absurd conceit of an illiterate. Symbolism returns to a mere sense-object's image. It is the experience of generating the thought-object corresponding uniquely to that tragedy as a composed, indivisible entity, not a

sensuous symbol, which is the referent for the idea of that drama taken as a whole.

That idea, that *Type* is the essential experience of the author, as composer, and of the audience in experiencing the discovery of this new thought-object, as one might regenerate an original scientific discovery, as a thought-object, in one's own, sovereign creative-mental processes.

The tragedy addresses so, implicitly, the central feature of all individual creative-mental activity; that central feature is the act of efficient *participation in humanity as an historical entirety.* Nicolaus of Cusa's elaboration of the principle of *capax Dei* references this impulse in its highest form of expression.[65] The *Types* associated with this creative impulse, include, most prominently, the following:

1. Man the individual as *imago viva Dei,* in *the living image* of *God the Creator.*[66] Man is thus set apart from, and above the beasts, by virtue of the fact that the successful existence of our human species is effected by creative activity of a *Type* centered upon the generation, transmission, and efficient assimilation of scientific and technological progress. Without this creative activity, mankind could not continue to exist as a human species. God's quality as *Creator,* and man's unique affinity to that God the Creator, is knowledge which depends upon a thought-object corresponding to this creative self-image of man.

2. *Man as the sovereign creative individual.* Although we are mortal, we exist efficiently in the present and whole future of all mankind by means of our employment of our creative-mental processes for the generation, transmission, and efficient assimilation of thought-objects *equivalent* to crucial features of scientific and technological progress. In this, every instance of generation of a thought-object, (whether an original discovery, or not) is a sovereign act of an individual person, rather than a "collective" effect.

3. The issue of creative discovery, is not resolvable in terms of case-by-case assessment of individual isolated such discoveries. The issue is the discovery and enhancement of an ordering-principle which directs us along a negentropic pathway of valid, successive discoveries. We require a process of valid discoveries. We seek a higher rate of this Type of growth of the rate of progress. In and of itself, the abstractly isolable, particular discovery by an individual person is of a transfinite order of lesser importance, than that person's contribution to improving the negentropy of that *ordering-principle of successive changes,* the which defines successive increases of potential population-density as a unified manifold. This latter consideration is the form of the most readily intelligible aspect of individual participation, not only in the classical tragedy, but also the universe as an entirety.

As is elaborated in other locations,[67] the individual affects efficiently, so, not only present and future generations, but also past. In the domain of space-time, in which the transfinite process of successive, negentropic *change* is ontologically the primary reality, this change is not merely the simple outcome of an individual act, but the outcome of participation in changing the universality of the determining process, the significantly efficient result of a person's mortal existence. Thus, we, by altering, through *participation,* the relevant feature of outcome of participation by even remote ancestors of the presently living generations, alter the past—by altering the outcome of the past's participation in the present and future.

Thus, in tragedy, Shakespeare causes the mind of Hamlet to be obsessed by what Hamlet believes to have been the ghost of his father. Thus, as by historical subjects of classical tragedy, do great poets seek to prompt their audiences to improve significantly the way in which we arrange the participation of the past in our present and future.

In this location, so far, we have emphasized those personalities whose very names are metaphors for the crucial thought-objects of scientific discovery. The pedagogical advantage of limiting our attention to such a selection of personalities, is that the work of discovery of these selected historical persons is readily susceptible to at least a negative form of mathematical treatment; on this account, the notion of a transfinite ordering of such discoveries through the issues of Cantor's *aleph*-manifold, is accessible.

Once the case is understood for such scientific metaphorizing, approximately at least, the concept is more readily extended to metaphor specific to classical art and statecraft. The favor is returned; from the extension to art and statecraft, we return the conception, much enriched, to scientific matters. The principal such enrichment is a keener sense, not only that valid science is essentially *subjective*—contrary to the popularized Cathar-Rosicrucian influences upon Descartes and British Empiricism; the meaning of science is not only Leibnizian physical economy, but, more broadly, contemporary man's efficient participation in the past, present, and future of the universe. We understand the essential role of classical art in making science possible, and understand the meaning of the metaphor; the highest form, the most rigorous form of mathematical physics is, thus,

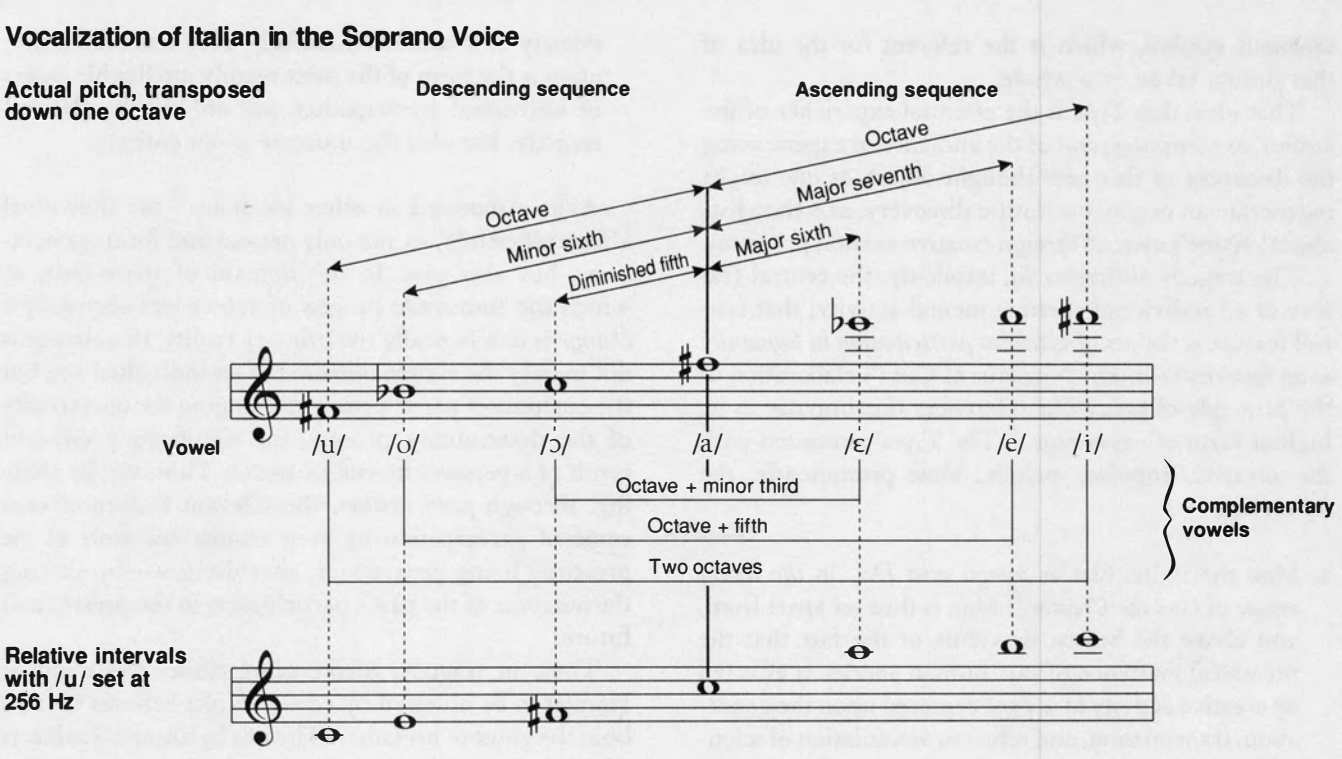


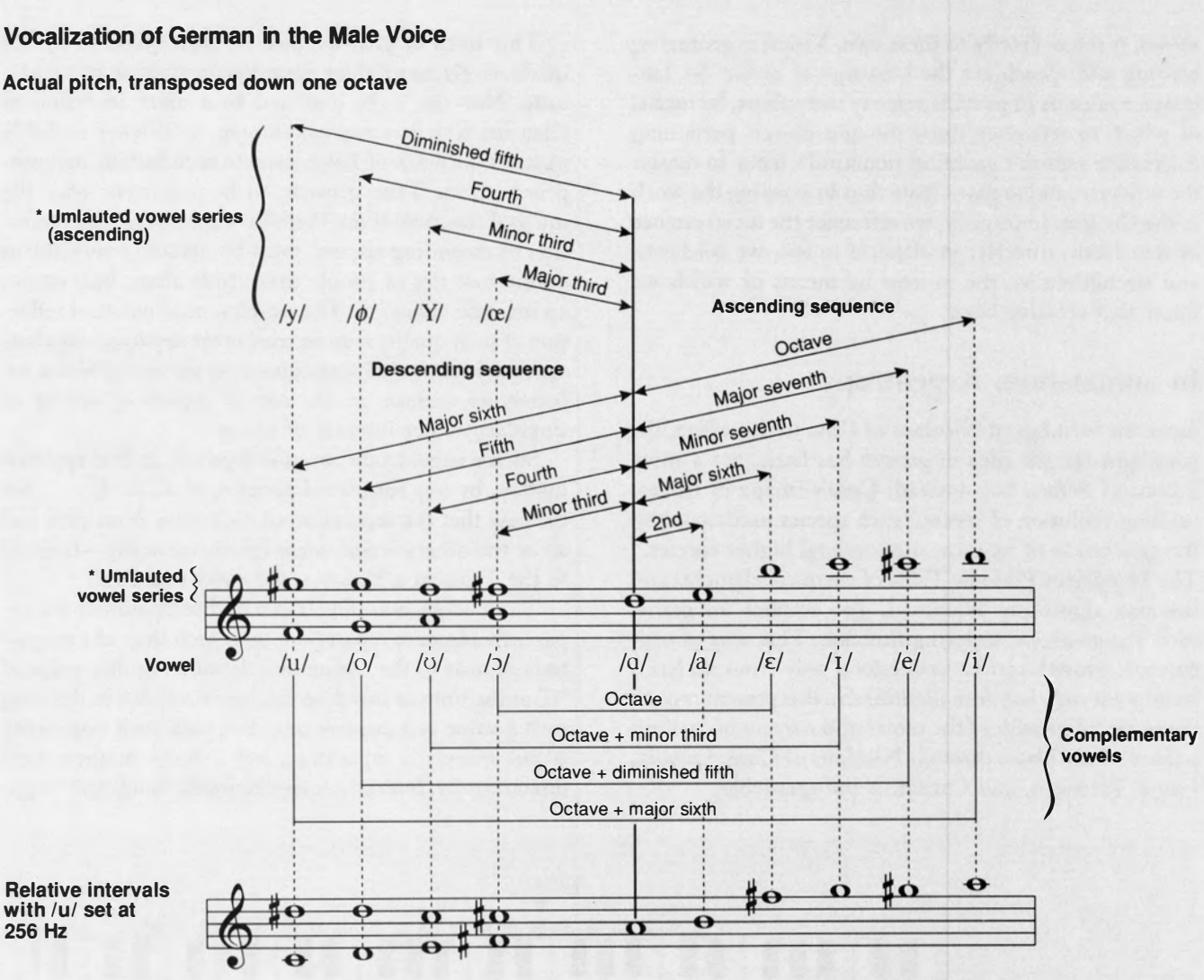
FIGURE 20. *The natural tuning of spoken language. The musical notes shown at the top of each chart as "actual pitch," are components of the sound of normal speech. The values emerge from laboratory measurements of frequencies (in Hz) of resonance peaks associated with the quality of each spoken vowel in the different languages, no matter at what fundamental pitch the vowel is spoken or sung. The vowels form a series of rising intervals from /u/—the darkest vowel quality—to /i/–the brightest. In order to illustrate the way in which intervals between the vowels are heard relative to each other, the notes at the bottom of the chart show the laboratory values transposed downward so that /u/ corresponds to C = 256 Hz. Values are shown for speakers of Italian and German.*

a body of classical poetry and music, a larger scope of classical philology derived from the polyphonic vocalization of the poetry.[68] Classical tragedy is part of this philology.

Musical Philology

As these sources identify the related points, the human singing—and speaking—voice has a natural set of characteristics and values which are shown clearly by the most efficient training and use of the vocal apparatus. That "most efficient" training and use, is the "Florentine *bel canto*" already in use no later than the middle decades of the fifteenth century, probably in the time of the great Cosimo de Medici's leadership there. "Most efficient" signifies the ratio of projected tone to air expelled from the singer's mouth and nose. The speaking of a language, notably the enunciation of the vowels (*vocalization*) is thus naturally *tuned* (SEE Figure 20).[69] Each natural species of adult singing (and speaking) voice has its own specific division among registers of mutually distinct "color," divisions located specifically (SEE Figure 21) within a specific between-note interval on the *C-256-pivoted*-well-tempered musical scale.

A relatively elementary illustration of the implications, is effected by attempting to compose a vocal quartet (soprano, mezzo-soprano, tenor, bass) in the medium of well-tempered polyphony, using an opening line from a classical poem such as, for English speakers, one by John Keats. First,[70] use the simplistic, but rigorous scheme of Goethe's favorite song-setter, J. F. Reichardt. Begin with the soprano part, setting the first utterance of the line within the soprano's second register. Then, examine the difficulties of writing a simple, four-part canon, copying the soprano part into each of the other three voices. A novice should try to copy with equal distance below the



second to third register shift for each species of voice.

Observe two of the most obvious features of these attempts. First, note in passing, the chords defined by the polyphony. Second, linger over the implications of the cross-voice sequencing. For an example of cross-voice sequencing, select a note from the bass line; read the note immediately following that in the tenor line; similarly, successively, from the mezzo-soprano and soprano lines. Repeat this cycle for each of the following tones in the bass line. Now, consider other cross-voice sequences, treating first all of the possible permutations which begin with the bass line. Note a similarity to some possible orderings within an *aleph*-manifold, as referenced earlier.

In each of these cross-voice sequences, observe the dissonances generated; but, do not end the matter there. Study the rules for classical canons from this standpoint. Generalize the notion of a resolution for each such dissonance. All of this is a process of the formation of a thought-object from the single polyphonic germ of one line of classical poetry. The fact that each dissonance implies a range of possible resolutions, defines a manifold of all of each. Thus, does a polyphonic setting of even a single line of classical poetry define implicitly a Cantor *Type*.[71]

The example just given is premised upon the mere rudiments of classical song-writing; yet it suffices to illustrate the notion, that music is the domain of metaphor, not of symbolism. Since this music originates in the naturally determined forms for polyphonic vocalization of classical poetry, the *transfinite essence* of musical composition must be recognized as an "hereditary" implication of classical poetry, and, thus, also of language in general, and drama.

In language, we have primary reference to the senses of vision and hearing; insofar as language references the

senses, it refers chiefly to these two. Vision is geometry; hearing and speech are the language of music. So, language equips us to provide sensory metaphors, by means of which to reference those thought-objects pertaining to creative reason's enabling mankind's labor to master the universe, and to participate thus in assisting the work of the Creator. In tragedy, we reference the social essence of that labor, directly; in classical music, we celebrate, and strengthen so, the process by means of which we foster that creative labor.

In Summation: Negentropy

Since the writings of Nicolaus of Cusa to this effect, the paradigm for the idea of *growth* has been, not a mere Fibonacci Series, but, instead, Cusa's image of the ascending evolution of species; each species participates in the generation of its own, superseding, higher species.[72] The Mendeleev Periodic Table of chemical elements and isotopes, rigorously examined, also implies integrally, such a negentropic ordering function. This idea of negentropic growth can be understood only from no less a standpoint than has been identified in this present report; the sweep of growth of the most valid current of modern science, from Plato, through Nicolaus of Cusa, Leibniz, Gauss, Riemann, and Cantor, is indispensable.

This form of growth must be understood to signify qualitatively more than mere linear increase of magnitude. Nor can it be confined to a mere inversion of Clausius-Kelvin statistical *entropy,* as Wiener foolishly misuses the work of Boltzmann to such banally inappropriate effect. True growth, to be consistent with the integral function of the Periodic Table, or Cusa's succession of ascending species, must be defined essentially as an increase not of simple magnitude alone, but, rather, an increase of *quality.* The simplest mathematical reflection of such quality is *an increase in the density of singularities (mathematical discontinuities) per interval of action,* or, better, *an increase in the rate of growth of density* of singularities per interval of action.

Such a negentropic series is depicted, in first approximation, by our functional series *A, B, C, D, E, . . .,* for the case that the separation of each term from each and all of the others is *equivalent* (mathematically—Cantor) to the *Types* of a higher-order *aleph*-manifold.

Those *aleph*-manifold *Types* of discontinuities are apparently absolute separations, and each thus of a magnitude as near to the notion of a definite number-value of "0" as the human mind, so far, has succeeded in defining such a value as a positive one. Yet, each such singularity is not merely a separation, not a mere mathematical discontinuity, but rather, an efficiently functional singu-

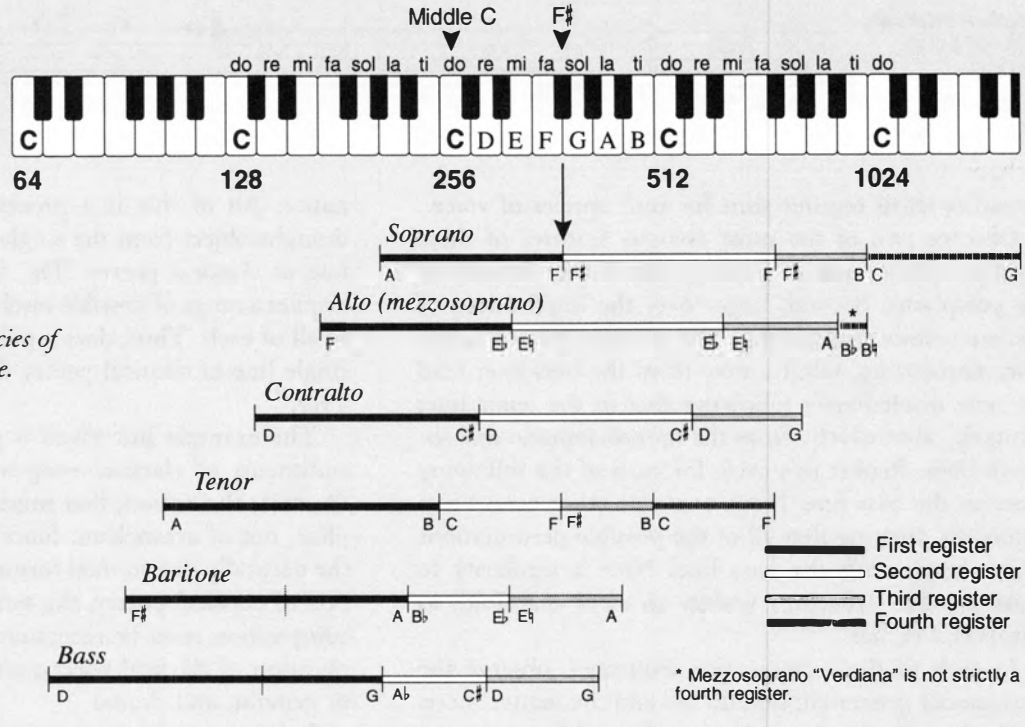


FIGURE 21. *The six species of the human singing voice.*

larity, whose content is equivalent to that of a thought-object—a *Monad*. That which Leibniz identifies as a *Monad,*[73] that toward which Riemann points with his *Geistesmassen,*[74] and that which is termed here a thought-object, has that functional significance.

Thus, we have situated the indispensable role of metaphor, as the essential poetic characteristic of any scientific or similarly rigorous communication. *Metaphor* is the key, the only possible means by which the unutterable is rendered *perfectibly intelligible* in communication among two or more persons.

As a matter of contrast, *symbolism* merely combines by reference, one sensory experience with another, or, in a worst case, the mere name for one thing with the name for another object or mere name. *Symbolism* is to intelligent communication as cabalistic numerology stands in opposition to both science and even mere sanity itself. Symbolism is merely combinatorial construction within the virtually empty domain of names.

An intelligent notion of *metaphor* hangs upon Plato's Socratic dialectic of negation. The referent is the experience of generating a true thought-object, not a sense-impression, through the processes of creative reason. Metaphor, so comprehended, is therefore the tactic by means of which two minds may coordinate an ordering among respectively similar thought-objects, in a problem-solving mode of creative thinking. This is the only means available to mortal persons, by which the unutterable thought-object is rendered, more or less adequately, *perfectibly* intelligible. Metaphor, so comprehended, is therefore the required essence of the secondary school classroom.

In fine art, the principle of metaphor is indicated, perhaps most sufficiently, by our rejection of "romanticism," "naturalism," and "modernism" in such art-forms as Classical music. Consider some selected highlights from the two-hundred-year history of Classical polyphony from the work of J. S. Bach through the 1890's Johannes Brahms.[75] Take two particular points of reference from within that domain: that revolutionary breakthrough in Classical polyphony effected by J. S. Bach's composition of his *Musical Offering,*[76] and what is identified as Joseph Haydn's discovery of the *Motivführung* principle of thoroughly integrated composition.[77] Examine these two as Wolfgang Amadeus Mozart combined their effect in his celebrated "Haydn" string quartets of 1782-1785.[78] This case, as continued by Beethoven, Schubert, and Chopin, illustrates the way in which all serious Classical musical composition is subsumed by the principle of metaphor.

The famous *Ricercar* of J. S. Bach's *Musical Offering* solves a problem in counterpoint by a tactic which Leibniz would recognize as *analysis situs*. Mozart's intensive, regular encounter with the work of Handel and the Bachs, at the regular Sunday, Vienna salon of Baron Gottfried von Swieten occurred during the time-frame Mozart was inspired by the celebrated "Russian" string quartets which Joseph Haydn had then just recently presented. The impact of Bach's *Musical Offering* is most striking in the sixth of Mozart's "Haydn" Quartets, the C-minor, "Dissonant," K. 465. The same connection is characteristic of Mozart's famous keyboard fantasy-sonata K. 475-457, which is quoted directly by Beethoven as his own keyboard sonatas Opus 13 and Opus 111, and also the C-minor violin sonata, Opus 30, No. 2. Beethoven's Opus 13 is quoted by Franz Schubert's posthumously published C-minor keyboard sonata; the opening movement of Chopin's "Funeral March" keyboard sonata quotes Beethoven's Opus 111. All of Mozart's major compositions of the 1782-1791 interval reflect his revolutionary insight into the combined importance of the two predecessors' cited discoveries.

The greatest representation of Haydn's *Motivführung* principle, is the "Credo" of Beethoven's *Missa Solemnis*. A beautiful, and masterful presentation of the same principle, is found in the opening movement of Brahms' *Fourth Symphony*. With the work of the 1782-1791 Mozart, the key to comprehension, and performance of each composition, is to locate the manner in which the *Motivführung* principle is elaborated to define the composition as a single, indivisible, unifying conception of the development of a single germ.[79]

The point of these brief references to Classical music, is to show how it is that all good Classical composition, especially since Haydn's referenced discovery, defines each thoroughly composed work as representing a single, integral, indivisible thought-object, a thought-object corresponding to a specific notion of *ordered development*.

The real music of such a Classical composition is a thought-object, for which the sensed aspect of the music is an indispensable metaphor. The thought-object appears "between the notes," so to speak, as the apparent, absolute mathematical discontinuities of the functional, non-linear series, *A, B, C, D, E, . . .*, taken as a whole, defines implicitly (*negatively*) the thought-object corresponding metaphorically to that series.

The most obvious of the discontinuities of a musical score, are the simple intervals defined by the time-intervals between tones, and by (*negatively*) duration of tones. The simplest notion of the ordering of intervals is a *scale* or *mode*. The changes from one to another scale or mode, are a higher ordering; and, so on.

These values are not relative values, but are situated with respect to an absolute, well-tempered scale of

C = 256 cycles, and are also situated with respect to vocalized poetic forms of speech, and, so forth and so on.

Therefore, the representation of that metaphorically situated thought-object, the which is the intent of the composition, requires rigorously clean polyphonic transparency. Differences must not arise except as differences are necessary to metaphorical representation of the relevant thought-object.

Thus, the performers must not simply perform the notes. They must, first, experience the relevant thought-object, and then read the notes to the purpose of causing the experienced dissonances and other differences in the performance to correspond to nothing but the metaphorical development of the unifying thought-object.

If we compare this overview of Classical music with Classical tragedy, seeking to grasp the common developmental characteristic of both media, we have a correct view of fine art, as Kant did not, a conception of art which corresponds to science as we have portrayed science here. If we comprehend the unity of a composition, one of any species of the fine arts, as being that composition's existence as a truly metaphorical work of art, and, if we seek out that conception of unity, as a precondition for our representation of that work of art, we are on the proper track.

A recent edition of selected Cantor correspondence contains a citation which is typical of Cantor's view of a certain important matter, and is directly relevant to the disgusting, and destructive incompetence of "information theory's" pretense to the name of "science":

> The majority of modern mathematicians, through the brilliant success of their self-perfecting formal character, which admits of more and more applications to the mechanical side of nature, have become flushed with a victory, which causes them to degenerate into materialistic one-sidedness and makes them blind to any objective-metaphysical knowledge and thus also to the foundations of their science.[80]

The root of that against which Cantor complains here, as he did frequently to the same effect in other locations, is the materialist tradition of "Buggery" as imposed upon seventeenth-century empiricism and Cartesianism by the Rosicrucian/Theosophist cult. This neo-paganist, materialist, "Enlightenment" cult, directed its energies toward uprooting and crushing the Christian Platonic tradition of Cusa, Leonardo da Vinci, Kepler, Leibniz, *et al.* Thus, it sought to uproot and destroy such specific, crucial thought-objects as Cusa's negative definition of the elementarity of circular action as universal least action, of Leonardo da Vinci's treatment of Golden Section harmonics, of Kepler's partition of elementary spherical space-time into negative and positive curvatures, of the seventeenth-century development of the interdependent notions of non-linear and least-action function.

As Cantor demonstrates, especially by aid of his richly historical treatment of his subject, driving non-algebraic function to the remotest boundaries of both macrocosm and microcosm, works to such effect that the Platonic principle of negation enables us to discover the necessary, intelligible existence of causal agency far beyond the furthest reach of non-algebraic function. Before this discovery could be made, it were necessary, not only to discover non-algebraic functions, but to show, from this vantage-point, that all ontological assumptions premised axiomatically upon an arithmetic or an algebraic standpoint are intrinsically false. Only by establishing such unique relative authority of non-algebraic function and its intrinsic, Leibnizian principle of universal least action, could the basis be found for discovery of the higher manifold.

If we today look back to Kepler's distinction, respecting harmonic implications, between positive and negative spherical curvatures, and note the derivation of modern "non-algebraic" function theory from such roots, we should recognize in this way why ignorance of the elementary discoveries of Cusa, Leonardo, Kepler, Leibniz, *et al.,* would blind modern victims of an empiricist education into seeing nothing but the mechanistic, entropic implications of positive curvature, being thus blind to the interrelated, dominant principles of negative curvature and least action. Hence, they cannot understand the nature of those limits of non-algebraic function upon which Cantor's most crucial discoveries rest.

What Cantor shows in this way cannot be compared, or contrasted to formal notions of function, in any ordinary sense. What Cantor demonstrates in fact, by the argument elaborated in his 1895-1897 *Beiträge*,[81] is that the formal aspect of the ordinary notion of mathematical function, even non-algebraic function, is but a metaphorical reflection of an entirely different ordering, an ordering of *thought-objects,* which order itself is, ontologically, also such a *thought-object.*

We enter thus into a world of such conscious objects, that their origins, their nature, their place, and their implicit effect, can be communicated to other minds; but, in this case, the conscious object—the thought-object itself—is unutterable in any mode of communication as such. In these cases, the communication of the object itself, from one mind to the other, occurs either by causing, dialectically, the creation of that other object in the mind of the hearer, or by prompting the hearer to recall such an earlier experiencing of the generation

of that thought-object.

That is also to say, that such communication cannot be effected as the transmission of "information"; but, rather, only by subordinating the process of communication to the most intensive and strict methods of Platonic, dialectical reasoning. Hence, all "information theory," insofar as it pertains to human thought, is not merely a fraud, but a monstrously destructive attack upon an entire crippled generation of victims. Unfortunately, under the evil influence of John Dewey and his like, and the more evil influence of the Frankfurt School and, now, the current, "New Age" and related reforms generally in effect in our classrooms today, that destructive fraud has become the hoax, which today passes for a more or less accepted standard for education.

Now, we conclude with closing words on the matter of the problem of intelligibility in the communication of thought-objects.

Refer, once more, to the pedagogical series of formal theorem-lattices, *A, B, C, D, E, ...* The intelligibility of both *A* and *B,* for example, as member-terms of such a non-linear function series, is found in the *change* of "hereditary principle"—of the axiomatic basis—which distinguishes *B* from *A.* The ontological quality of this function of *change* is located formally "within" the *aleph*-manifold *Type* of discontinuity between each pair of terms. That *change,* so formally located, is the causal feature of the process as a unified whole. The equivalent aspect shared among all such changes in that series, defines a *Type*, and also defines a thought-object corresponding, as a *One,* to the generation of the *Many* terms of this series.

This aspect of the matter is expressed in the communication-process by the Platonic form of *negation* of the "hereditary," axiomatic principle separating one set of underlying assumptions—as for theorem-lattice *A*—from all other sets of a series—such as theorem-lattice *B,* or *C,* or *D, ...* Communication in this Platonic mode, as employed, for example, by Cusa, is the only possible *Type* of communication of those conceptions—thought-objects—which are not susceptible of explicit representation within the linear "band-pass" of any medium of communication itself.

The essential feature of all such Platonic communication is predominantly twofold. First, the essential thought-object, to which all other thought-objects should be referenced, is the notion of *negentropy* as that has been implicitly, metaphorically defined here (that takes into account "anti"-negentropy). Second, that the reality to which our thought-object manifold's *Type* must correspond, is mankind's successfully negentropic social reproduction of our species—*imago viva Dei*—in the universe, by our negentropically-ordered changes in mankind's practice upon that universe.

It must be in art as in science. *Truthfulness* is Socratic irony, and *Truth* is a metaphor. We cannot say what we mean; but, we can render our conceptions of, and intent *to change* intelligible to other minds, by aid of a rigorous regard for the fact that information so-called is never more than metaphor.

NOTES

1. Cervantes' *Don Quixote* is a Platonic form of classical tragedy given in prose form. See Miguel de Cervantes, *The Ingenius Gentleman Don Quixote de la Mancha,* trans. by Samuel Putnam (New York: Random House, 1949).
2. William Empson, *Seven Types of Ambiguity* (Middlesex: Penguin Books, 1961).
3. Cf. Bernhard Riemann, "Zur Psychologie und Metaphysik," on Herbart's Göttingen lectures, for Riemann's reference to *Geistesmassen,* in *Mathematische Werke,* 2nd. ed. (1892), posthumous papers, ed. by H. Weber in collaboration with R. Dedekind.
4. Archimedes, "Measurement of a Circle" and "Quadrature of the Parabola," in *The Works of Archimedes,* ed. by T.L. Heath (New York: Dover Publications). Cardinal Nicolaus of Cusa, *De Docta Ignorantia* (*On Learned Ignorance*), Book I, trans. by Jasper Hopkins as *Nicholas of Cusa on Learned Ignorance* (Minneapolis: Arthur M. Banning Press, 1985); also, "De Seculii Quadratura" ("On the Quadrature of the Circle"), trans. into German by Jay Hoffman (Mainz: Felix Meiner Verlag).
5. See G.W. Leibniz, "History and Origin of the Differential Calculus," in *The Early Mathematical Manuscripts of Leibniz,* trans. by J.M. Child (LaSalle: Open Court Publishing Co, 1920), pp. 22-58.
6. It is always useful in mathematics, to reflect upon the physical implications of one's calculations. At $n = 112$, two adjacent polygonal angles on the circumference of a circle of 1-centimeter radius are 1.21009×10^{-33} centimeters. For a $n = 256$ polygon, 5.42626×10^{-77} centimeters. Thus, to increase the distance between two adjacent angles of an $n = 256$ polygon to a significant 10^{-33} centimeters, would require a circle 2.23006×10^{43} larger than our circle of a 1-centimeter radius. The radius of this larger circle would be 2.236006×1038 kilometers, or 2.35717×10^{25} light years. Compare this kind of calculation with Archimedes' famous sand-reckoner ("The Sand-Reckoner," in *The Works of Archimedes, op. cit.*). How old is a universe whose radius is 2.35717×10^{25} light years?
7. Plato, *Parmenides,* Loeb Classical Library (Cambridge: Harvard University Press); Lyndon H. LaRouche, Jr., "Project A," in *The Science of Christian Economy and Other Prison Writings* (Washington, D.C.: Schiller Institute, 1991).
8. Plato, *Timaeus,* trans. by R.G. Bury, Loeb Classical Library (Cambridge: Harvard University Press, 1975); see also *Plato's Timaeus: The Only Authentic English Transtlation,* trans. by associates of Lyndon H. LaRouche, Jr., *The Campaigner,* Vol. 13, No. 1, Feb. 1979.
9. Luca Pacioli, *De Divina Proportione,* for which Leonardo da Vinci did the drawings. See, among various articles in the volume, Augusto Marinoni, "The Writer: Leonardo's Literary Legacy," in

The Unknown Leonardo, ed. by Ladislao Reti (New York: McGraw-Hill Book Company, 1974), which includes plates of Leonardo's drawings in *De Divina Proportione,* for this, as well as later references to Leonardo's hydrodynamics.

10. Johannes Kepler, *Harmonice Mundi, (On the Harmony of the Worlds*), in *Opera Omnia,* vol. 5, Frankfurt (1864), of which only Book V has been translated into English, in the *Great Books of the Western World* series (Chicago: Encyclopaedia Britannica, Inc.); *On the Six-Cornered Snowflake,* trans. and ed. by Colin Hardie (Oxford: Clarendon Press, 1966), reprinted by *21st Century Science & Technology,* 1991. See LaRouche, "In Defense of Common Sense," chap. VIII, and "The Science of Christian Economy," Appendices I-III, V, VI, in *Christian Economy, op. cit.;* also *A Concrete Approach to U.S. Science Policy,* chap. II (Washington, D.C.: Schiller Institute, 1992).
11. Pierre de Fermat, *Oeuvres Fermat,* ed. 1891, epistl. xlii, xliii. Christiaan Huygens, *The Pendulum Clock, or Geometrical Demonstrations Concerning the Motion of Pendula as Applied to Clocks,* trans. by Richard J. Blackwell (Ames: Iowa State University Press, 1986). Gottfried Wilhelm Leibniz, "Specimen Dynamicum" (1695), in *Leibniz Selections,* ed. by Philip P. Wiener (New York: C.S. Sons, 1951); also, *Système Nouveau de la Nature,* 1695. Johann Bernoulli, *Acta Eruditorum,* May, 1697; trans. in D.E. Smith, *A Source Book in Mathematics* (New York: Dover, 1959), pp. 648-55. Jakob Bernoulli, *Acta Eruditorum,* May, 1697; trans. in D.J. Struik, ed., *A Source Book in Mathematics, 1200-1800* (Princeton, N.J.: Princeton University Press, 1986), pp. 292-299.
12. Johannes Kepler, *Snowflake, op. cit.*
13. See LaRouche, *U.S. Science Policy, op. cit.,* chaps. II and III.
14. *Ibid.*
15. *Ibid.*
16. If the argument here appears to parallel that of Georg Cantor on *aleph* types, and Kurt Gödel's famous proof, the resemblance is intentional and well-grounded. See Georg Cantor, *Beiträge zur Begründung der transfiniten Mengenlehre* (*Contributions to the Founding of the Theory of Transfinite Numbers*), trans. by Philip E.B. Jourdain (New York: Dover Publications, 1955). See also Ernest Nagel and James R. Newman, *Gödel's Proof* (New York: New York University Press, 1958).
17. See LaRouche, "In Defense of Common Sense," chaps. II-IV, in *Christian Economy, op. cit.*
18. *Ibid.*
19. Cf. Nicolaus of Cusa, *De Docta Ignorantia, op. cit.,* Book I, chap. XIII, which has the use of a very large circle to approximate a straight line.
20. Gilles de Roberval, "The Cycloid," from *Treatise on Indivisibles,* trans. Evelyn Walker (New York: Teachers College, 1932); also, quoted in D.J. Struik, *op cit.*
21. Christiaan Huygens, *The Pendulum Clock, op. cit.*
22. *Ibid.*
23. Christiaan Huygens, *The Pendulum Clock, op. cit.* Johann and Jakob Bernoulli, *op. cit.* Gaspard Monge, *Application de l'algèbre a la géométrie,* 5th ed., ed. by J. Liouville (Paris: Bachelier, 1850).
24. Christiaan Huygens, *Treatise on Light* (1690), trans. Sylvanus P. Thompson (New York: Dover Publications, 1962). Gaspard Monge, *op. cit.*
25. G.W. Leibniz, "On Analysis Situs," in *Gottfried Wilhelm Leibniz Philosophical Papers and Letters,* trans. and ed. by Leroy E. Loemker (Chicago: University of Chicago Press, 1956), Vol. I, pp. 390-396. Bernhard Riemann, "Lehrsätze aus der Analysis Situs . . .," in *Mathematische Werke, op. cit.,* quoted as "On Analysis Situs," in D.E. Smith, *op. cit.*
26. Johannes Kepler, *Snowflake, op. cit.* See also LaRouche, *U.S. Science Policy, op. cit.,* chap. IV.
27. Eugenio Beltrami's devastating refutation of the entire theory of elasticity upon which the Maxwell electromagnetic theory is based can be found in "Sulle equazioni generali dell' elasticita" ("On the General Equations of Elasticity"), *Annali di Matemàtica pura ed applicata,* serie II, tomo X (1880-82), pp. 188-211; trans. by Rick Sanders, *21st Century Science & Technology,* unpublished.
28. Georg Cantor, *Theory of Transfinite Numbers* (*Beiträge*), *op. cit.;* see also, *Georg Cantors Gesammelte Abhandlung,* ed. by Ernst Zermelo (Hildschein, 1962). Also, *Gödel's Proof, op. cit.*
29. Like the philosophically allied project, the "Frankfurt School" of Adorno, Horkheimer, Marcuse, Heidegger, Arendt, *et al.* (see Michael J. Minnicino, "The New Dark Age: The Frankfurt School and 'Political Correctness,' " *Fidelio,* Vol. 1, No. 1, Winter 1992), modern linguistics was also launched by the 1920's Communist International. The key Communist official was Stalin's collaborator in this project, Germany's Karl Korsch. During the 1930's, Korsch collaborated on this project with Rudolf Carnap, both in turn collaborating with Bertrand Russell and the Russell-Hutchins "Unification of the Sciences" project, in the initial, pre-war sessions held at the University of Pennsylvania. The University of Pennsylvania's Professor Harris adopted this linguistics as his profession, followed by his student, today's Professor Noam Chomsky.
30. See Plato, *Timaeus, op. cit.;* Euclid, *The Thirteen Books of Euclid's Elements,* Books 10-13, trans. T.L. Heath (New York: Dover Publications, 1956); Luca Pacioli, *De Divina Proportione;* Johannes Kepler, *Harmonice Mundi, op. cit.,* chap. II; and also, Leonhard Euler, "Elementa doctrinae solidorum," St. Petersburg Academy of Science (1751).
31. We pass over, for the moment, the additional stellated solids defined by, first, Johannes Kepler, *Harmonice Mundi, op. cit.,* chap. II; and Louis Poinsot, *Memoirs sur Les Polygons et Les Polyhedras* (*Notes on Polygons and Polyhedra),* trans. by Laurence Hecht, *21st Century Science & Technology,* unpublished.
32. Leonardo of Pisa (Fibonacci), *Liber Abaci* (*The Book of the Abacus*), as quoted in D.J. Struik, *op. cit.*
33. See footnote 11 for the relevant works of Huygens, Leibniz, and the Bernoulli brothers.
34. Christiaan Huygens, *Treatise on Light, op. cit.* Cf. Johann Bernoulli, "Curvatura Radii," in Diaphanous Nonformabus *Acta Eruditorum,* May, 1697, as quoted by D.J. Struik, *op. cit.,* pp. 391-399.
35. See LaRouche, *U.S. Science Policy, op. cit.,* chap. III.
36. *Ibid.* Leibniz's commentary on this view of Newton is in his first letter to Clarke, from 1715: "Sir Isaac Newton and his followers have also a very odd opinion concerning of the work of God. According to their doctrine, God Almighty wants to wind up his watch from time to time; otherwise, it would cease to move." In Clarke's reply, he acknowledges that God "not only composes or puts things together but is himself the author and continual preserver of their original forces or moving powers." Reprinted in *Leibniz Philosophical Papers, op. cit.,* Vol. II, pp. 1095-1169.
37. In 1850, Rudolf Clausius wrote his first article discussing the theory of heat. Clausius' book was without experimental proof, and also without any reference to a "universal law." In 1852, Wiliam Thomson (later Lord Kelvin), wrote an article entitled, "On a universal tendency in nature to the dissipation of mechanical energy." This article consisted of ideological speculations on the experimental work on heat-powered machines of the French scientist Sadi Carnot, in which Thomson had not participated. In that article, Thomson postulated that the universe, since it was nothing but a machine, would one day run down. In 1854, Thomson's friend Helmholtz used the same thesis in his *On the Transformation of Natural Forces.* Finally, Clausius, in the second (1865) edition of his book, after a meeting with Thomson, concluded the book with the famous two axioms: (1) the energy of the universe is constant; and (2) the entropy of the universe tends toward a maximum. See LaRouche, *U.S. Science Policy, op. cit.,*

chap. III.

38. See LaRouche, *U.S. Science Policy, op. cit.,* chap. III.
39. A postulate is distinguished as an unproven assumption, in imitation of a true axiom, introduced to eliminate arbitrarily an otherwise ambiguous or incomplete feature of the theorem-lattice manifold generated by a pre-existing set of axioms and postulates.
40. The great Hilbert is reported to have thrown student Norbert Wiener out of a scientific seminar at Göttingen for persistent methodological incompetence.
41. Cf. Winston Bostick, "The Plasmoid Construction of the Superstring," *21st Century Science & Technology,* Vol. 3, No. 4, Winter 1990; also, "How Superstrings Form the Basis of Nuclear Matter," *21st Century Science & Technology,* Vol. 3, No. 1, Jan-Feb 1990.
42. Hence, as Philo of Alexandria denounced the *gnostic* followers of Aristotle among Jewish rabbis of his time, neither Creation nor God could actually exist in a nominalist form of deductive system such as that strictly attributable to Aristotle. See Philo, "On the Account of the World's Creation Given by Moses" ("On the Creation"), in *Philo,* Vol. I, trans. F.H. Colson and G.H. Whitaker, Loeb Classical Library, (Cambridge: Harvard University Press, 1981).
43. Works in which Cusa emphasized the isoperimetric principle include *De Docta Ignorantia, op. cit.,* Book I, and "On the Quadrature of the Circle," *op. cit.*
44. St. Augustine, *De Musica* (*On Music*), trans. by R. Catesby Taliaferro (Annapolis: St. John's College Bookstore, 1939).
45. LaRouche, "In Defense of Common Sense," chaps. II-IV; and "The Science of Christian Economy," chaps. II-IV, VI; in *Christian Economy, op. cit.*
46. Cf. Nicolaus of Cusa. Cusa's view in an early work, *On Learned Ignorance,* is that "God has implanted in all things a natural desire to exist with the fullest measure of existence that is compatible with their particular nature . . . . There is in them a discernment that is natural and in keeping with the purpose of their knowledge, which ensures their natural inclination serving its purpose and being able to reach its fulfillment."

    Later, in "The Vision of God" (1464), Cusa develops the conception that each species, with its natural faculties as they develop, "yearns" for the existence of a higher species, as man does for the knowledge of the Absolute, of God. Here, Cusa's idea of negentropic species-evolution as the characteristic of Creation, is expressed by the poetic conception *terminus specie.* The universe consists of negentropic growth of higher orderings, whose microcosm is human reason. The species recognizes this divine order of creation, in its own way, and becomes a singularity in the transition from one ordering to the next. Thus the species has a *terminus specie,* the actualization of infinity in one point, which enables further development. "This power, which I have from Thee, and in which I possess a living image of the power of Thy almightiness, is the free will through which I am capable of either increasing or reducing the capacity to recive Thy beneficence."
47. See LaRouche, *U.S. Science Policy, op. cit.,* chap. III, section "The Geometry of Creative Reason."
48. See LaRouche, "In Defense of Common Sense," chaps. IV, XII; "Project A," chaps. XVII, XVIII; and "The Science of Christian Economy," chaps. IV, VII; in *Christian Economy, op. cit.*
49. On Descartes' *deus ex machina,* see LaRouche, *U.S. Science Policy, op. cit.,* chap. IV.
50. Sir Isaac Newton, in his *The Mathematical Principles of Natural Philosophy* (New York: The New York Philosophical Society, 1964), states "*hypothesis non fingo*" (I don't make hypotheses), and explains his reasons for this on grounds of *induction* versus hypothesis.
51. See LaRouche, *U.S. Science Policy, op. cit.,* chap. IV.

    We first hear of the Bogomils in the tenth century A.D. in Bulgaria. (In Bulgarian, *Bogomil* means "beloved of God.") Among their beliefs is the characteristically gnostic one, that the Father of Jesus Christ was not the Creator of the world. For the Bogomils and later the Cathars, the power of the devil worked through the nature and constraints of the material world; matter and spirit were never meant to cohabit. This division and its corresponding principles of good and evil, light and darkness, is broadly called dualism. For the origins of the Bogomil or Cathar cults in Manicheanism, and the Albigensian Crusade against them, see LaRouche, *Christian Economy, op. cit.,* pp. 485-486.
52. The Cathar cult was known in France as the Bulgarian cult, or "Les Bougres," which translated into English as "the Buggers." Because of the cult's peculiar sexual perversion, which flowed from their gnostic doctrine of separation of matter and spirit, it resorted to various other kinds of sexual activity, and thus the name "Bugger" became associated in English with homosexuality.

    Overt gnostic cultism continues to this day, including the sexual perversions. In Colombia, for example, the head of the Universal Christian Gnostic Church, Samael Aun Weor, is the author of a book entitled *Perfect Marriage,* which asserts: "The age of sex is coming, the New Age of Aquarius . . . . Sexual magic will be officially admitted in the universities of the new Aquarian Age." The book continues: "To create a child, you do not need to spill semen. The spermatozoid which escapes without spilling semen is a choice spermatozoid of a superior nature, totally mature. The result of such impregnation is a new creation of exremely high order. That is how we can form a race of Supermen. In the mysteries of Eleusis, the sacred dances, the naked dances, the burning kiss and sexual connection, they make men unto Gods . . . the Sufi dances and the whirling dervishes are tremendiously marvelous." Aun Weor is also the author of *The Social Transformation of Society,* which sketches the Gnostics' political program for Latin America. The Gnostic Church has been the political controller of the M-19 narcoterrorists who today share power with the government of Colombia.
53. See Friedrich Schiller, "The Legislation of Lycurgus and Solon," in *Friedrich Schiller, Poet of Freedom,* Vol. II, ed. by William F. Wertz, Jr. (Washington, D.C.: Schiller Institute, 1988).
54. See Fred Henderson, "Free Trade, The Confederacy, and Slavery," *The New Federalist,* Vol. V, No. 36, Nov. 11, 1991, pp. 5-6; "The Lee myth is debunked but not the more dangerous mythmakers," *Executive Intelligence Review,* Vol. 18, No. 38, Oct. 4, 1991, p. 62ff.
55. The decrees of the Roman Emperor Diocletian (284-305 A.D.) attempted to freeze the economic crumbling of the Roman Empire by fixing prices and wages by law. This led in the fourth century to the reforms of the Emperor Theodosius, which established legal enforcement of the occupation which each Roman citizen was forced to follow for his entire life. These Malthusian reforms were the earliest attempt to impose socialist decrees by totalitarian government. See *Global Showdown,* §2.3 (Washington, D.C.: Executive Intelligence Review, 1985), on the edicts of Diocletian and his successors; see also Kenneth Kronberg, "How the Romans nearly destroyed civilization," in *The Genocidal Roots of Bush's 'New World Order'* (Washington, D.C.: Executive Intelligence Review, 1992), pp. 158–59.
56. See, for example: Benjamin Franklin, "Proposals Relating to the Education of the Youth in Pennsylvania," Philadelphia (1749). Thomas Jefferson, "A Bill for the More General Diffusion of Knowledge" (1779), in *Thomas Jefferson: Writings,* ed. by Merrill D. Peterson (New York: Library of America, 1984): "[T]he most effectual means of preventing [tyranny] would be, to illuminate, as far as practicable, the minds of the people at large . . . . [Therefore] it becomes expedient for promoting the public happi-

ness that those persons, whom nature hath endowed with genius and virtue, should be rendered by liberal education worth to receive, and able to guard, the sacred deposit of the rights and liberties of their fellow citizens, and that they should be called to that charge without regard to wealth, birth, or other accidental condition or circumstance." John Adams, "Thoughts on Government" (1776), in *American Political Writing During the Founding Era: 1760-1805,* Vol. I, ed. by Charles S. Hyneman and Donald S. Lutz (Indianapolis: Liberty Press, 1983). Benjamin Rush, "A Plan for the Establishment of Public Schools and the Diffusion of Knowledge in Pennsylvania; To Which Are Added, Thoughts upon the Mode of Education, Proper in a Republic" (1786), in *American Political Writing, op. cit.*

57. See Friedrich Schiller, "Aesthetical Lectures (1792-1793)" and Wilhelm von Humboldt, "On Schiller and the Course of His Spiritual Development," both in *Friedrich Schiller, Poet of Freedom, op. cit.* Humboldt, who predicated his work on the influence of and education provided him by Schiller, was for a time responsible for all educational policy in Prussia.
58. See Immanuel Kant, *Critique of Pure Reason,* trans. by Norman Kemp Smith (New York: St. Martin's Press, 1965); *Critique of Practical Reason,* trans. by Lewis White Beck (Indianapolis: Bobbs-Merrill Company, 1956); also, in particular, *Critique of Judgment,* trans. by J.H. Bernard (New York: Hafner Press, 1951), §30-54, p. 152ff.: "[Genius] cannot describe or indicate scientifically how it brings about its products . . . . [A] Homer . . . cannot show how his ideas . . . come together in his head, simply because he does not know, and therefore cannot teach others."
59. Plato's arguments connecting the idea of the Good (or the Absolute Infinite as expressed by later Christian Platonists), both to the evolution of the physical universe, and to the process of Becoming proper to human reason, are developed with more and more arduous rigor in a number of dialogues: *Theaetetus, Parmenides, Sophist, Republic, Philebus, Timaeus, Critias.*
60. See footnote 58.
61. Immanuel Kant, *Critique of Judgment, op. cit.*
62. See *A Manual on the Rudiments of Tuning and Registration,* Vol. I (Washington, D.C.: Schiller Institute, 1992), chap. 11.
63. See footnote 1.
64. Georg Cantor, *Theory of Transfinite Numbers, op. cit.*
65. See Nicolaus of Cusa, "On Conjectures," in *Philosophisch-Theologische Schriften,* Vol. II (Vienna: Herder & Co., 1982), p. 158. "Man is indeed God, but not absolutely, since he is man; he is therefore a human God. Man is also the world, but not in a contracted way everything, since he is man; man is therefore a microscosm or a human world. The region of humanity therefore embraces God and the whole world in its human potentiality."
66. See Nicolaus of Cusa, "On the Filiation of God," in *Philosophisch-Theologische Schriften, op. cit.,,* p. 640. "Indeed, just as God is the actual essence of all things, so is the intellect, separated and united in itself vitally and reflexively, a living similitude of God. Therefore, as God Himself is the essence of all things, so the intellect, the similitude of God, is the similitude of all things. Cognition, however, is effected through similitude. However, since the intellect is an intellectual living similitude of God, it knows, when it knows itself, everything in itself as the one."

    See also Philo of Alexandria, *op. cit.,* §XXIII: "Moses tells us that man was created after the image of God and after His likeness (Gen. 1:26). . . . Let no one represent the likeness as one to a bodily form; for neither is God in human form, nor is the human body God-like. No, it is in respect of the Mind, the sovereign element of the soul, that the word 'image' is used; for after the pattern of a single Mind, even the Mind of the universe as an archetype, the mind in each of those who successively came into being was moulded. . . . [The human mind] opens by arts and sciences roads branching in many directions, all of them great highways. . . . [W]hen on soaring wings it has contemplated the atmosphere and all its phases, it is borne yet higher to the ether and the circuit of heaven, and is whirled round with the dances of planets and fixed stars, in accordance with the laws of perfect music, following that love of wisdom which guides its steps. And so, carrying its gaze beyond the confines of all substance discrenible by sense, it comes to a point at which it reaches out after the intelligible world."
67. See LaRouche, "In Defense of Common Sense," chap. XI; "Project A," chap. II; and "The Science of Christian Economy," chap. V; in *Christian Economy, op. cit.*
68. Cf. *A Manual on Tuning, op. cit.* References are the Preface: "The Classical Idea," *passim*; chap. 2: "The Six Species of Singing Voice"; chap. 9: "The Principles of Bel Canto"; chap. 10: "The Synthetic Geometry of Composition"; and chap. 11: "Artistic Beauty: Schiller vs. Goethe."

    See also LaRouche, "Solution to Plato's Paradox of the 'One and the Many,' " and Jonathan Tennenbaum, "The Foundations of Scientific Musical Tuning," *Fidelio,* Vol. 1, No. 1, Winter 1992.
69. See *A Manual on Tuning, op. cit.,* chaps. 9 and 10.
70. Cf. *A Manual on Tuning, op.cit.,* chap. 11.
71. The attribution of musical notions to Cantor's work is ironically most appropriate. Cantor was an able amateur musician, of a musical tradition traced to his maternal grandfather *Kapellmeister* Ludwig Böhm, whose violinist brother, Joseph, was the teacher of the great virtuoso Joachim. (Adolf Frankel, *Das Leben Georg Cantors,* cited in *Georg Cantors Gesammelte Abhandlung, op. cit.,* p. 452.) It was this Ludwig Böhm who delivered the definitive performance of Beethoven's late string quartets on Beethoven's behalf.
72. See footnote 46 for Cusa's concept of species-evolution.
73. See, G.W. Leibniz, *Monadology,* trans. by George Montgomery (LaSalle: Open Court Publishing Co., 1989).
74. See footnote 3.
75. The, unfortunately, popularized myth of an "Hegelian" division of musical history, into successive "baroque," "classical," and "romantic" periods, should be simply ignored as nonsense. The work of *Classical composers* such as J. S. Bach, his famous sons, Haydn, Mozart, Beethoven, Schubert, Mendelssohn, Schumann, Brahms, *et al.,* is separated by *a moral principle of composition* from the contrasting, irrationalist principle of ascending chromatic eroticism adopted by such nineteenth-century *Romantics* as Berlioz, Liszt, and Wagner, *et al.*
76. Johann Sebastian Bach, "Musical Offering," BWV 1079 (New York: G. Schirmer, 1944).
77. Joseph Haydn, Opus 33, "Russian" string quartets (Mineola, N.Y.: Dover Publications, 1985), ed. by Wilhelm Altman.
78. Cf. Bernhard Paumgartner, *Mozart* (München: 1991), chap. 31, pp. 299-311; p. 548.
79. See *A Manual on Tuning, op. cit.,* chap. 12 *passim,* on the principled approach of Beethoven and Brahms to composing a set of variations on a theme.
80. Cited in Herbert Meschkowski and Winfried Nilson, eds., *Georg Cantors Briefe,* (Heidelberg: Springer Verlag, 1991), pp. 9-10, 478; from J. Bendiek, "Ein Brief Georg Cantors an Pater Ignazius Jeiler O.F.M.," *Franzisch Kannischer Studien* 47, 1965, pp. 65-73. "Die modernen Mathematiker in ihrer Mehrheit durch den glänzenden Erfolg ihres stets sich vollkommenden Formalwesens, das immer mehr Anwendungen auf die mechanische Seite der Natur zulässt, in einen Siegesrausch hineingeraten sind, der sie zur materialistischen Einseitigkeit verkommen lässt and sie für jegliche objektiv-metaphysische Erkenntnis and daher auch für die Grundlagen ihrer Wissenschaft blind macht."
81. See footnote 28.